Balancing Students' Valuing And Mathematical Values

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The students' influence and responsibilities on content and learning processes are important objectives emphasised in all steering documents for Swedish education. However, results from a large-scale survey exploring what students find important for learning mathematics show that students may not value such openness in mathematics teaching and learning. We found that aligning teaching to students' valuing would rather conserve a tradition of teacher authority. In this discussion essay, these results will be related to the obstacles teachers may experience when fulfilling educational objectives of students' responsibility, participation, and influence on the planning of teaching and learning mathematics.

Conflicting Values in Swedish Mathematics Education

The two most important things for learning mathematics, due to Swedish students, responding to the WiFi-survey (see below), are:

1. Explaining by the teacher
2. Knowing the times tables

These results left us with questions about the importance teachers should assign to students' valuing of activities for learning mathematics. With the result that Swedish students value teacher explanations and times tables as the two most important activities when learning mathematics, how do these answers relate to values? And how will such values allow students' influence on the planning and evaluation of teaching, as stipulated in the curriculum?

We will, in this paper, describe how mathematical values were used as an analytical tool for better understanding tensions between students' valuing when learning mathematics and statements in the

Swedish mathematics education steering documents. Values/valuing act on different levels in the networking practises of mathematics education (Valero, 2004). In order to learn more about students' values in the Swedish mathematics classroom, we participated in an international values-survey, the WiFi-survey (What I Find Important, when learning mathematics) (Seah & Wong, 2012), which will be used as a background of the coming discussion. WiFi is a quantitative web survey, in Sweden distributed to 750 respondents in grade five and eight (eleven and fifteen-year old students). The participating students were asked to value 64 items, consisting of activities from the mathematics classroom, by marking their importance for learning mathematics on a scale, from absolutely unimportant to absolutely important (Andersson & Österling, 2013). However, having used mathematical values (Bishop, 1988) as an analytical tool, we realized that values in different parts of the network were in conflict. Our aim in this paper is to discuss how teachers' possibilities to fulfil curricular goals of student influence on teaching in mathematics are affected by such value conflicts.

Values in Curriculum and Classroom Culture

The Swedish educational system is not value-neutral. It grants students a large amount of influence and responsibility, as being part of civil democratic obligations. Students' right to influence and their responsibilities are clearly stated in the recent Swedish steering documents (Skolverket, 2011, p. 17):

Teachers should:

- take as their starting point that the pupils are able and willing to take personal responsibility for their learning and work in school,
- be responsible for ensuring that all pupils can exercise real influence over working methods, forms and contents of education, and ensure that this influence grows with increasing age and maturity…
- together with the pupils plan and evaluate the teaching, and
- prepare pupils for participating and taking responsibility, and applying the rights and obligations that characterise a democratic society.

Swedish mathematics classrooms can be regarded as cultures consisting of stable values. The steering documents emphasize the importance of general values. Here we focus on influence, participation, and responsibility. Inculcation of the values stated in the objectives, and values of mathematics, is part of entering a mathematics community, and thereby part of learning mathematics. However, since values often are obscure for the participant in a cultural group (Hofstede, Hofstede, & Minkov, 2010), it is difficult to relate events in the classroom to values. Thus, we did not regard values as something that could be derived from direct observations of classroom practices. Instead, we accepted Bishop's mathematical values (1988) as an analytical tool that allowed us to categorise our observations. Together with knowledge of cultural practices in Swedish community (c.f. Andersson & Österling, 2013) and in the Swedish mathematics classroom, this analysis allowed us to discuss underlying factors in order to explain teachers' and students' choices. Bishop (1988) has introduced the concept of "mathematical values" with the purpose to understand the choices of teachers and students in the mathematics classroom. Three pairs of opposing values describe three dimensions of values in mathematics, and the table below is an attempt to briefly present those mathematical values:

Table 1
Mathematical values (Bishop, 1988)

DIMENSION	PAIRS OF OPPOSING VALUES	
Ideological values: the ideology of mathematics	*Rationalism* – reasoning and argument is valued	*Objectism* – symbolising and applying ideas of mathematics
Sociological values: who can do mathematics	*Openness* – mathematics is democratically open for anyone to use and explain	*Mystery* – fascination and mystique of mathematical ideas and their origin
Sentimental values: What sensations mathematics can bring	*Control* - a sense of certainty and power through mastery of rules	*Progress* – the sense of ideas growing through questioning

These categories were used as an analytical tool for describing values in the different networking practices, from steering documents and classroom culture, to the view of the child in Swedish culture.

The aim of the steering documents, to allow and underline the importance of students' influence on the planning and evaluation of mathematics teaching, can be expected to align well with the valuing of *openness* in mathematics teaching. Information about mathematics is easily accessible for students to make use of, since no one "owns" the Pythagorean theorem or other mathematical objects. Thereby, students should be able to use and chose mathematics content from a number of sources, if openness is valued in the mathematics classroom.

Thus, to what extent does mathematics lend itself to such negotiation? The sentimental mathematical value of control focuses certainty and power through the mastery of rules (Bishop, 1988). Thereby, having access to the formula of the Pythagorean theorem will not automatically render the correct use of it. The authoritative nature of mathematics through its "interest in certainty" is central (Wagner & Herbel-Eisenmann, 2013). This authoritative nature is related to the valuing of control (Bishop, 1988). In this way, mathematics is epistemologically different from other subjects in school, since deductive reasoning based on already stated axioms, rather than empirical explanations and students own initiatives, are valued. Thereby, arriving at students' influence on the planning of teaching may be challenging in mathematics. We present three examples from Swedish research, which demonstrate teachers' efforts to meet the expectations of students' influence and responsibilities.

First, a study made by Hansson (2011) shows that the responsibility for learning in Swedish classrooms too often is passed on from teachers to students and their textbooks (Johansson, 2006), by expecting students to work on their own in textbooks. Teachers, in the name of individualized or student-centred learning, too often abandon students who need their guidance. Hansson (2011) showed how especially second language learners suffered in this type of teaching. Instead of improving inclusion in mathematics education, this way of making students "responsible" for their learning widens the gap between students who are familiar with the expectations and discourses within the mathematics educational culture, and students who do not have access to the particular classroom culture. However, there is nothing in the way these teachers chose mathematical activities that can be related to the valuing of openness in mathematics. It could be the case that students are expected to take responsibility in a classroom culture of mystery, where the origins of mathematical ideas and procedures are hidden for students.

Our second example is a study by Lingefjärd and Maier (2010), where interactions between secondary students and their teachers in a mathematical modelling process are compared. A modelling process can be expected to be an open activity, where students develop mathematical models for describing real world phenomena. In one classroom, a group of students wants to discuss their formula with their teacher, who responds: "Well, if it is your formula, then go ahead and explain it!" (p. 103). When researchers asked further about this interaction, the teacher quoted the Swedish steering document (Skolverket, 1994), which states that students should learn to "work on their own". However, it became obvious that these students neither benefitted, learned mathematics, or to work independently from their interactions with their teacher. They did not have access to the knowledge required for taking responsibility for learning; it rather became evident that allowing students to take responsibility will not take all responsibility away from the teacher. Wagner & Herbel-Eisenmann (2013) discuss both the authoritative nature of mathematics, and how teachers, as being the more knowledgeable, need to handle authority at different classroom levels:

> It is important to be *an* authority in mathematics and to be in authority to some extent as a teacher, but it is also important to establish a routine in which each student sees him/herself as *in authority* of his/her own learning so that s/he too could become *an authority* in mathematics" (Wagner & Herbel-Eisenmann, 2013, p. 8).

In the examples above, the teachers resigned as authorities before the routine of students being authorities of their learning was established.

In our third example, we discuss a research project where Swedish upper secondary students became agents for their learning through taking responsibilities for the planning of mathematical projects and assessment, and thereby fulfilled objectives in Swedish curriculum (Skolverket, 1994, 2011), again concerning students' influence and participation. Andersson (2011) worked as a researcher together with Elin, a mathematics teacher. The valuing of openness regarding the mathematics content aligned with the aim of allocating students responsibilities for planning and implementations of the projects. Through working consciously in this particular way, initially not recognized by students as part of the mathematics classroom culture, a change developed over time. Recognition is a prerequisite for change and formation of a new discursive practice (Gee, 2011). Thereby, until this personal responsibility is recognised as part of the mathematics classroom, it must be acknowledged that preparing students to participate and take their responsibility needs both time and practice.

The three examples above illustrate how students' influence on teaching might be challenging for mathematics teachers. Students do not automatically benefit and learn more mathematics when responsibilities are negotiated. Students' possibilities for achieving influence and acting in a responsible way are facilitated if students understand and agree on the stated objectives. Mathematics learning objectives may focus the mathematical content, but it may also focus mathematical values. Among the mathematical values, in our examples above, valuing openness seemed important when the teachers' intention was to allow students' responsibility for learning.

Students' Values: Some Results from the Wifi-Survey

The underlying assumption in the WiFi-study is that values guide students' decisions about what is important for learning mathematics. Such values may be mathematical values, general educational values, or cultural values. Mathematical values can be defined as "the deep affective qualities which education fosters through the school subject of mathematics" (Bishop, 1999, p. 2). Values being "deep" suggest that there may be an unawareness of the values held by a person, or values in one's own culture. It is easier to become aware of values in a foreign culture, than to see your own culture's values (Hofstede, Hofstede, & Minkov, 2010). When "education fosters" values, there therefore may be an unawareness of possible alternatives. The fostering of mathematical values hence becomes part of learning mathematics, as enculturation. In grade five or eight, the enculturation and fostering of mathematical values has been going on for five or eight years respectively. Students' answers must therefore be understood as reflecting values of Swedish mathematics classrooms, rather than a metacognitive valuing of the importance of an item for learning mathematics (Andersson & Österling, 2013). Considering the results we got, aligning teaching to what students value as important seems to render a rather "traditional way", or in Skovsmose's (2001) words, teaching within an "exercise paradigm" with a teaching based mainly on teacher instructions, teacher authority and drill learning. Thus, the result we received in the WiFi-study rather raises questions about how students' valuing may improve more inclusive and democratic forms of teaching.

The data analysis from the WiFi-study was made through calculating means and standard deviations. The respondents valued each item on a scale from "Absolutely important" to "Absolutely unimportant". This scale was transferred to a Likert-scale, where "Absolutely Important" was assigned the value 1, and "Absolutely Unimportant" the value 5. "Explaining by the teacher" was most important, with a mean of 1.33 and a standard deviation of 0.70. The second most important item was "Knowing the times tables", with the mean of 1.43 and standard deviation of 0.77. These two results are used as illustrative examples for a discussion of how students' responses can be related to mathematical values. We posed three questions relating to each of the items:

1. How do students understand the question?
2. What can we say about the mathematical knowledge related to this question?
3. How does the question relate to mathematical values?

We start with the second most important item, "Knowing the times tables". First, how do students understand the question? Mathematics teaching does not exist in a vacuum. It is affected by, and at the same time affects, cultural expressions. For example, the Swedish children's books about Pippi Longstocking, a nine-year old very strong girl (first published in 1945) may serve as an illustrative example. Pippi lives on her own without any caretaker. However, her two (well-behaving) friends tell her she needs to attend school. This scene describes the *very first* (and only) time Pippi ever enters a classroom:

> "Hey, everybody," hollered Pippi, swinging her big hat. "Am I in time for pluttification?" (Lindgren, 2007, p. 60)

Pluttification tables, or the multiplication tables, as a properly fostered student would put it, are well known by Swedish students. Swedish children's literature contains a theme of children challenging authoritative structures (Aronsson & Sandin, 2014), and even though Pippi is a fiction she captures the importance of the multiplication tables, and at the same time challenges the motives for school and school mathematics. The Pippi-books are known by (almost) all children in Sweden, and this example illustrates how Swedish children recognise times-tables as being part of the school mathematics. Therefore, we have concluded that students answer according to what they usually do in mathematics, rather than how important it is with respect to their mathematical learning.

Second, what can we say about the mathematical knowledge related to this question? The predominant way of treating times tables in Swedish mathematics classrooms is to automatize the knowledge. Hence teachers may ask children to "Practice until you know the answer even when someone wakes you up in the middle of the night". One explanation might be found in Swedish schooling history. Lundin (2008) outlines the history of Swedish mathematics education, where there were two curricula until 1968. One school curriculum was intended for working-class children, expected to learn mathematics as a skill useful in their future profession. Bourgeois children attended an academic mathematics curriculum, where for example Euclidean Geometry had an important role. Even nowadays, after the school has been united for 45 years, there are still tensions between the utilitarianism of mathematics and the academic view in the steering documents. Calculating skills, as knowing the times tables, still seems to be valued mathematical knowledge.

Third, how do "knowing the times tables" relate to mathematical values? Depending on how the teaching of times tables is enacted, different values will be in focus. Valuing rationalism would be, for example, to focus explaining why $7 \times 0 \neq 7$, whereas $7 + 0 = 7$. Valuing the opposite, objectism, is focusing to learn times tables by heart, as a mathematical object. Another value that could relate to times tables is control, the mastery of rules. From the discussion above, we will consider "knowing the times tables" as related to objectism and control, rather than the opposing values of rationalism or progress.

We now pose the same three questions to the most valued item: "Explaining by the teacher". First, how do students interpret the question? We cannot know if students value the explaining of the teacher when someone raises the hand, individually at the desk, or if it is the teacher explaining in a whole-class discussion. In either case, the teacher is expected to possess mathematical knowledge that can be transmitted to students by explaining.

Second, what can we say about the mathematical knowledge related to this question? Mathematics seems fixed, as always containing a correct explanation and a correct answer, and what education does is to allow students to take part of this fixed knowledge. Questions that would allow students to question or explore mathematics, such as "Investigations" or "Making up my own math questions", are valued less important than "Explaining by the teacher" (the differences in means are significant at the 5% level).

Third, how does "Explaining by the teacher" relate to mathematical values? Striving for a correct explanation or answer relates to the value of control. It could be a way of valuing rationalism over objectism, assuming that the explanation contains questions that asks students to reason, or that the explanation provides examples of reasoning. Both forms can be found in Swedish classrooms (Björklund Boistrup, 2010). It may also relate to the value of mystery over openness, since the origin or explanation of mathematics remains obscure. The teacher might know, and must be the one explaining to students rather than opening up mathematics through questioning.

To sum up, the results presented here demonstrate how Swedish students primarily value control and objectism in mathematics learning. Openness is not valued as equally important. Now we ask ourselves, how can values be part of students' influence and participation in mathematics education?

Values and Fostering

It seems unlikely that students' valuing of "knowing the times tables" would reflect a valuing of the importance of times tables for their learning of mathematics. The activities students' value due to the WiFi-survey seems instead to be in line with the teaching they have experienced from prior school years. Therefore, students valuing of "teacher explaining" and "knowing the times tables" is interpreted rather as a reflection of how mathematics teaching in Sweden often has been conducted. Using mathematical values (Bishop, 1988) as our analytical tool allows us to uncover how values at different levels of the networking practices of mathematics education sometimes are at conflict. We could relate students' valuing to the values of control and objectism, and we have shown that these results align with what is common practice in Swedish mathematics classrooms and in Swedish culture (Aronsson & Sandin, 2014; Lindgren, 2007; Lundin, 2008). Hence, students answering the WiFi-survey have been told by Pippi, siblings, parents, and teachers that times-tables are important for learning mathematics.

The steering documents (Skolverket, 2011) focus student influence, responsibilities, and participation in planning of teaching and their learning, which align with the mathematical valuing of openness. However, students value items that relates to openness as less important according to our results from the WiFi-survey. A contradiction occurs where openness in our curriculum invites students to influence teaching, and students chose not to value openness. The WiFi-survey reveals that aligning students valuing of activities for learning mathematics results in conserving a traditional way of teaching mathematics, focusing objectism and control, or the mastery of rules and right answers. Therefore, teachers who intend to foster another view of mathematical knowledge, such as rationalism or openness, will encounter difficulties, when students on the one hand are valuing the opposite, and on the other hand are allowed to "exercise real influence over working methods, forms, and contents of education" (Skolverket, 2011, p. 19).

As presented above, there seem to be at lest two strategies teachers use to obtain students' influence; they either abandon their authority and expect students to be responsible for their learning (Hansson, 2011; Lingefjärd & Maier, 2010,), or they try to empower students by offering learning experiences where students have agency for deciding on context, evaluation, and assessment (Andersson, 2011). The fostering of mathematical values takes place in the mathematics classroom, in contrast to general values, that are fostered in different cultural contexts (Bishop 1988). It is the mathematics teacher who introduces the students to the culture of mathematics, and thereby to the mathematical values. Two of the examples discussed (Andersson, 2011; Lingefjärd & Maier, 2010) deal with modelling and project work, a mathematical content that easily lend itself to students' influence on planning, and offers possibilities for openness. However, in Lingefjärd & Maier (2010), students seem to have too little mathematical knowledge to be able to solve the task. Thus, working independently and being responsible for learning does not exclude the importance of a knowledgeable teacher. Thereby, teachers earn their authority as being more knowledgeable in mathematics. Students may be invited to be an authority of their learning (Wagner & Herbel-Eisenmann, 2013). However, this process urges a recognition of mathematics as not being an authoritative subject, but an open subject, allowing learners to participate by posing questions and choosing how to use or explore mathematics.

In the research accounted for in this paper, the concept of values has been useful as an analytical tool for understanding conflicts between steering documents and the culture and traditions in the mathematics classroom. We argue that before expecting students to take responsibility for learning, the mathematical values needs to be addressed and negotiated by teachers and students. The valuing of mathematical openness seems important for this purpose. Discussing values in mathematics education brings us all the way back to fundamental questions like what is the nature of mathematics, and what is the purpose of teaching mathematics in school. Indeed, a lot is yet to be explored about the role of values, in the process of fostering students' to become participants of the mathematics culture, as well as for obtaining student influence and responsibility in learning mathematics.

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