Building a Case for Understanding Relational Dimensions in Mathematics Classrooms

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Research on mathematics instruction often focuses on issues of problem solving, explanations, and discussions. However, relational aspects of classrooms may be just as important, particularly in understanding the success or failure of underserved students. The paper briefly looks across four studies that examine dimensions of relational interactions in mathematics. This research details a case study, builds a framework for understanding relational dimensions of mathematics classrooms, and uses regression to study links with student achievement. The work builds the case that relational dimensions of classrooms are critical in understanding student learning and engagement with mathematics.

In examining instruction in urban schools, mathematics education research often focuses on dimensions such as problem solving, mathematical discussion, student explanations, and cognitive depth. However, this focus on content instruction may overlook relational dimensions that impact learning. We think there is much to gain in attending to this as a mechanism that impacts student learning in mathematics. As a field, we may be underestimating the impact of instruction on student learning when not considering mechanisms such as relational dimensions in the mathematics classroom (Lubienski, 2002). While quality content instruction is necessary, it may be insufficient in generating the kinds of student understandings that the field aims for, particularly for underserved students. We begin by reviewing literature on relational dimensions of mathematics classrooms and then briefly discuss four related studies building a case that relationships are a critical element for the field to examine in more detail.

Teacher-Student Relationships in Mathematics

Scholars have approached the study of teacher-student relationships in mathematics in a number of ways. For example, Hackenberg (2010) builds off of Nel Nodding's (1984) work on an ethic of care to understand how teachers can form caring relationships with students, both with respect to their mathematical ideas and their emotions. In working with four students, she shows how teachers can build caring relations through mathematical support. Bartell (2011) adds to this work by conceptualizing "caring with awareness", which explicitly addresses cultural and racial aspects of relationships. In theorizing caring with awareness, she considers that to care for many Latino and African American students, who are often marginalized in the US educational system, teachers must take on student perspectives. In doing so, teachers take on explicit stances that challenge stereotypes about who is mathematically competent. Across this work, caring for student contributions, emotions, and cultural backgrounds are central.

Cultural backgrounds are also critical in attending to behavior. African American and Latino students endure more conflictual relationships and behavioral discipline than their white peers (Jerome, Hamre, & Pianta, 2009). In general, research shows that teachers overreact when addressing behaviors of African American and Latinos, placing a strong emphasis on physical control (Monroe, 2005). Monroe (2005) determined that teachers with a limited understanding of students' cultures tended to issue severe behavioral sanctions. Even when controlling for economic status, Gregory and Weinstein (2008) found that teachers interacted more negatively with African American and Latino students by issuing sanctions. How teachers address student behaviors is critical then in teacher-student relationships.

Boaler and Empson highlight the importance of attending to student contributions in mathematics. Boaler (2006) documented teachers that highlight the intellectual value of student contributions through explicit statements, questioning, or asking students to share their mathematical thinking. Through these teacher actions, students were framed as competent mathematically, disrupting the low status and fixed notions of ability that are common in mathematics. Empson's (2003) research also speaks to the importance in framing students' contributions as having value. She found that for the lowest

achieving mathematics learners, positive interactions with the teachers served to enhance their mathematical identity and performance. Both of these scholars highlight how acknowledging the mathematical ideas of learners framed students as mathematically competent.

Gorgorió and de Abreu use social representations to understand teacher-student interactions (Gorgorió & de Abreu, 2009). In this work, they highlight the way in teachers dismiss different ways of thinking mathematically or misinterpret various cultural representations within mathematics. Similar to Bartell, this work on social representations highlights cultural aspects of mathematics classrooms that impact student engagement. In contrast, Civil (2007) focuses on classrooms where teachers value the cultural knowledge of parents and students. In blurring the boundary between the school and home, teachers valued the everyday practices that students brought to mathematics. These scholars emphasize cultural perspectives as central to understanding students' representations and knowledge.

Finally, Setati and Adler draw attention to the overlap between culture and language (Setati, Adler, Reed, & Bapoo, 2002). In their research, code-switching serves as both a tool to move between informal and formal talk as well as across mathematical discourses. Relatedly, Moschkovich (2002) highlights the importance of language in bridging relationships with students and how language can construct mathematical competence. She discusses the importance of giving students access to mathematical discourse, defined more broadly than vocabulary. These researchers highlight the complexity in supporting students' language practices, their cultural nature, and implications for mathematics learning.

This work raises the complexities involved in teacher-student relationships within mathematics including dimensions such as acknowledging student contributions, providing emotional support, highlighting student competence, and attending to linguistic and cultural resources. However, this work does not look across these relational dimensions. The studies discussed below extend this research by examining relational interactions across dimensions.

Study 1: A Case Study of Relational Interactions

This case study of a 4th grade classroom examined mathematics instructional quality and teacher-student relational interactions. We define relational interactions as a communicative action or episode between teachers and students, occurring through verbal and non-verbal behavior that conveys meaning (Battey, 2013). In considering "good teaching" in mathematics, scholars usually refer to teacher content knowledge and instruction that promotes understanding (Wilson, Cooney, & Stinson, 2005). However, these two elements are often not as prevalent in urban contexts, a space where high percentages of African American and Latino students are educated (Lubienski, 2002). Using video, field notes, and an interview, the case study found an urban teacher that engaged students in substantive mathematics, but a number of relational interactions seemed to disrupt access to mathematics. Across two classroom lessons, the study found four dimensions of relational interactions that mediated access to mathematics: addressing behavior, framing mathematics ability, acknowledging student contributions, and attending to culture and language (Battey, 2013). The two research questions were: 1) How does an elementary teacher exhibit mathematics knowledge and instructional practices in her classroom? 2) How do relational interactions shape access to mathematics for African American and Latino students?

The teacher, Ms. Spencer, displayed quality content instruction across the lessons. She moved students to more sophisticated numbers and encouraged generalizations using a variety of pedagogical strategies. Sometimes her relational interactions were aligned with these mathematical goals. Her positive interactions encouraged student strategy use, affirmed students' mathematics ability, and connected the mathematics to familiar contexts. On the other hand, a number of interactions contrasted with her mathematical goals. In these negative episodes, Ms. Spencer isolated students, questioned students' ability, ignored student thinking, used sarcasm, withheld instruction, and focused on language issues to the detriment of mathematics learning. What is particularly interesting in this case is that relational interactions were mostly orthogonal to the quality of content instruction.

This case study speaks to the fact that our conceptions of "good" mathematics teaching are often too narrow, particularly for

underserved students. What seems missing in Ms. Spencer's interactions and comments is a deeper understanding of why these episodes take place. Her relational interactions enable or restrict access to quality mathematics, regardless of the form of instruction provided. When she praises students, affirms student ability, and encourages them to go deeper into the mathematics, we see a teacher who is using relationships to support the practices we so often seek in urban mathematics classrooms. When Ms. Spencer uses sarcasm, withholds instruction, treats students as invisible, and misses student contributions, we see a teacher who may be holding deficit views of students. But research in mathematics education does not have a framework to bring more understanding across relational dimensions.

Study 2: A Framework for Relational Interactions

The second study extends the first by developing a framework for relational interactions across seven 4th and 5th grade classrooms in one urban school. To further this line of research, the study aimed to look at the association between relationship interactions and instruction across classrooms. The two research questions were: 1) What are the types, frequency and intensity of relational interactions in elementary mathematics classrooms? and 2) How do relational interactions relate to the quality of mathematics instruction?

A fifth dimension of relational interactions was identified in this study, setting the emotional tone (Battey & Neal, resubmitted). This dimension builds on Hackenberg's (2010) work around attending to students' emotions. The interactions that constituted this additional dimension spoke to broad messages that teachers passed on to students, sometimes centering on the need to struggle through mathematics or affirming multiple ways to practice mathematics. Additionally, interactions such as framing mathematics ability and attending to culture and language, while infrequent, sent intense messages to students. These infrequent dimensions are ways in which teachers reproduce or challenge broad discourses about who can or cannot engage mathematics.

Addressing behavior and acknowledging student contributions were more frequently occurring dimensions though the former was more negative and the latter positive (see Table 1). The frequency

Table 1
Relational Interactions: Dimension, Frequency, and Intensity

DIMENSION	TEACHER							ALL
	Ms. B	Mr. J	Mr. D	Mr. L	Mr. G	Ms. S	Mr. T	
Behavior								
Positive	2(1.0)[a]	0	0	2(1.0)	0	0	1(1.)	5(1.0)
Negative	1(1.0)	0	3(2.7)	23(1.8)	0	6(1.7)	18(1.9)	51(1.9)
Ability								
Positive	0	0	0	2(1.0)	6(2.5)	2(2.5)	2(1.5)	12(2.1)
Negative	2(3.0)	0	2 (2.0)	0	0	2(2.5)	1 (2.0)	7(2.4)
Contributions								
Positive	5(1.6)	4(1.3)	6(2.0)	3(1.7)	8(2.1)	18(1.5)	11(1.2)	55(1.6)
Negative	11(2.5)	3(1.3)	3(2.3)	2(2.0)	0	19(2.2)	1(1.0)	39(2.2)
Culture								
Positive	0	0	0	1(2.0)	4(2.3)	2(1.0)	0	7(1.9)
Negative	0	0	0	1(1.0)	0	1(3.0)	0	2(2.0)
Tone								
Positive	0	3(1.0)	0	0	3(1.7)	0	0	6(1.3)
Negative	0	0	0	0	0	4(1.0)	0	4(1.0)
Total								
Positive	7(1.4)	7(1.1)	6(2.0)	8(1.4)	21(2.2)	22(1.5)	14(1.2)	85(1.7)
Negative	14(2.6)	3(1.3)	8(2.4)	26(1.8)	0	32(2.0)	20(1.9)	103(2.0)
Interactions/ minute	0.43	0.31	0.44	0.71	0.75	0.55	0.74	0.56

[a] *Average intensity*

and intensity of the interactions differed significantly across the
classrooms, as did the quality of instruction. Rates of relational inter-
actions varied between .30-.44 per minute in classrooms with more
traditional instruction to around .70 per minute in classrooms with
more reform-oriented instruction. This means almost twice as many
interactions occurred in more reform-oriented classrooms, which
are largely based on discussions. This increased rate provided more
opportunity to acknowledge student contributions – a dimension
that was more positive than negative. However, the increased rate of
interactions sometimes meant that rates of negative interactions in

these classrooms were higher as well. The implication is that while mathematics educators might try to increase reform-oriented content instruction in classrooms, this potentially could increase negative relational interactions as well.

The increase in interactions raises a critical issue of changing behavioral expectations as instructional norms shift in classrooms. When teachers take on more reform-oriented practices, as they were supported in doing during this study, teachers may struggle with communicating behavioral expectations for students. An often-overlooked issue in supporting teachers as they change instruction is how this change makes it necessary to communicate different behavioral expectations and therefore requires teachers to develop different strategies for managing students. While this may not seem like an issue for mathematics educators, if behavioral issues result in school discipline (see Gregory & Weinstein, 2008), and in turn removes a student from instruction, it heavily impacts mathematical access.

Interestingly, despite differences in instruction and rates of relational interactions, no relationship existed between the quality of relational interactions and instruction for these seven teachers. The finding that relational interactions do not necessarily parallel instructional quality raises the need for future research.

Study 3: Successful Relational Interactions

The conceptualization of relational interactions as within instruction allows for a better conceptualization of what constitutes high-quality mathematics instruction, particularly for urban African American and Latino students. To that end, this study identified teachers who succeeded with students based on high student performance, quality content instruction, and building strong relationships (Battey, Neal, Leyva, & Adams-Wiggins, under review). Using video data of seven 2nd and 3rd grade teachers, the study explored one research question: How do urban elementary teachers, who successfully support their students, engage in relational interactions within their mathematics classrooms? The study offers a key contribution to the literature by detailing strong teacher-student relationships in urban elementary mathematics classrooms within one district.
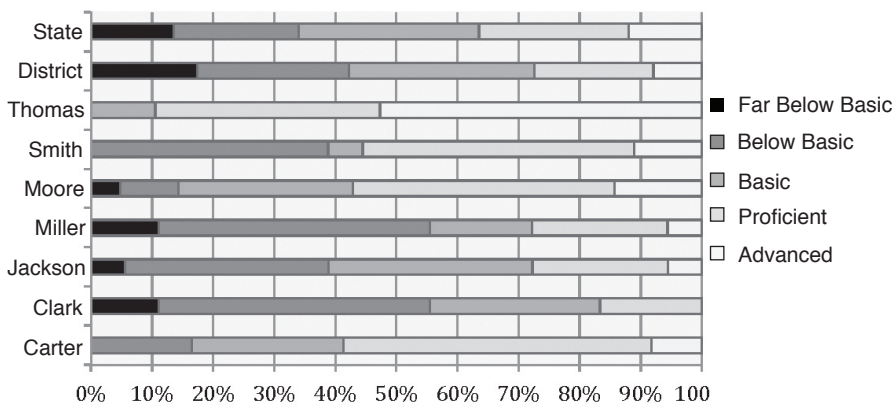
Figure 1: Achievement Levels of Classrooms on the CST

Quantitative data from state assessments and classroom video were used to select successful teachers. After selecting two teachers (see Thomas and Moore in Figure 1) based on achievement data, instructional quality, and positive relational interactions, the analysis detailed the relational interactions between these teachers and their students. Almost 90% of Thomas' students achieved at the proficient or advanced level on the state achievement test, with no students scoring at the two lowest levels. Her students performed statistically better than students in all of the other classrooms except for Moore's. Moore's had over 20% more students achieve proficiency than the state and they performed statistically better than all but Carter's students. All of the teachers scored high on instructional quality, but only Thomas, Moore and Jackson had overall positive relational interactions with students. Therefore we detailed Thomas and Moore's relational interactions.

A number of patterns were evident in unpacking Ms. Moore and Ms. Thomas' relational interactions in the mathematics classroom. Both teachers made their expectations clear for students' mathematical engagement with peers. Additionally, Ms. Thomas consistently recognized positive models of behavior and neither teacher escalated episodes when noting students' off-task behavior. Both teachers pushed beyond answers, requiring justification and consistently pressing for more complete explanations. Ms. Moore was particularly adept at extracting the important mathematics even when students had incorrect answers. Ms. Moore and Ms. Thomas framed students as competent in contrast to stereotypes of the mathematical abilities

of African American and Latino students. Ms. Moore also actively incorporated student-generated mathematics problems, giving students more ownership of the content. Ms. Thomas, on the other hand, openly showed vulnerability with her students during instruction and praised students for their thinking and fluency with the mathematics.

Consistent with study 2, some of the teachers in this study had an extensive negative focus on behavior. However, Moore and Thomas were balanced in addressing student behavior. In fact, both teachers noted positive behavior more than misbehavior. Additionally, they set clear expectations for how students should engage with each other in the mathematics classroom. This is noteworthy given that this study was performed in the last quarter of the school year and teachers were still restating their expectations to students.

Table 2
Frequencies and Average Intensity of Relational Interactions

TEACHER	DIMENSION									
	Behavior		Ability		Contribution		Culture		Tone	
	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.	Pos.	Neg.	Pos.	Neg
Moore	16(1.31)[a]	13(1.31)	2(1.5)	0	24(1.88)	3(1.33)	2(1.0)	0	9(1.56)	0
Thomas	7(.143)	4(1.25)	7(1.)	0	35(1.4)	4(1.25)	0	0	3(1.67)	0

[a] Average Intensity

Another connection between this study and the extant literature are the caring mathematical relationships that Bartell (2011) conceptualized. The lessons did not provide clear examples of how either teacher drew on culture or language in instruction besides Ms. Moore drawing on student-generated mathematics problems. It is possible that if more lessons had been videotaped, more examples of this dimension may become apparent. However, both teachers shared power in the classroom and exhibited elements of care in terms of assigning competence to students that are typically negatively stereotyped mathematically. The design of this study does not allow the authors to say whether or not these were intentional in challenging racial narratives, but they certainly run counter to the deficit narratives that are so pervasive about mathematics achievement among students of color. The study offers a key contribution to the literature by detailing strong teacher-student relationships in urban elementary mathematics classrooms.

Study 4: Relational Interactions and Achievement

Study four examined how teacher-student relational interactions affect students' mathematics achievement. Analyzing the seven 2nd and 3rd grade teachers and their 137 students in study 3, this paper explored two research questions (Battey & Leyva, 2013): 1) How do relational interactions explain variance in student achievement in elementary mathematics classrooms? and 2) How do relational interactions differ based on sex and ethnicity?

We entered the relational interactions dimensions as independent variables with the state achievement test as the dependent variable into a linear regression. Across all of the students, the only significant relationship was Acknowledging Student Contributions (F = 21.57, p < .01). It explained 13.4% of the variance in students' mathematics test scores. Since only the third graders took the test the prior year, a linear regression analyzed third grade scores with the added predictor of prior achievement. Prior achievement accounted for 61.4% of the variance in students' subsequent scores. The only other variable that accounted for a significant part of the variance was setting the emotional tone, accounting for 12.6% of the variance in scores. The effect size for the model was high (F = 63.63, p < .01).

Teachers engaged in acknowledging student contributions at a higher rate for females than males across both grades (p < .01). This raises concerns about whether the mathematical contributions of Latino and African American boys were missed in classrooms. It is not surprising that attending to culture and language and framing mathematics ability did not produce any significant results. This was probably due to two issues: (a) the infrequency of these relational interactions and (b) tendencies to engage in these dimensions with the whole class rather than with individual, resulting in a lack of variance.

These findings point to the impact of teachers' roles on student achievement by way of attending to students' mathematical ideas. The results also speak to a needed reconceptualization of mathematics instruction as both an academic and social mechanism affecting equitable opportunities.

Conclusion

The approach of detailing specific relational interactions builds on prior work that looks at dimensions such as language, emotion, and culture within mathematics classrooms. From a case study showing how relational interactions can be orthogonal to content instruction to links with change in achievement, the need to examine relational dimensions of mathematics classrooms grows. We have been intentional in making a case, building a conceptual framework, using the framework to elaborate practice, and linking interactions with student achievement. In looking across relational dimensions, a focus on interactions holds potential in adding to our understanding of access to mathematics for different student populations. For one, looking at relational interactions for various student groups could create comparative contexts to see if teachers' attitudes about groups change the frequency and intensity of interactions. For instance, do African American and Latino students experience more negative relational interactions than their white peers? This might explain a piece of the puzzle in terms of classroom mechanisms that impact student learning. While this area of research is still developing, we think it provides a way to codify observable classroom behaviors to tease out the relationships that students are building with teachers and the mathematics.

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