The Role of Rational Dependence in the Mathematics Classroom

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This conceptual piece defines rational dependence, or the reasoned dependence on the knowledge of others, and provides a rationale for its inclusion in the mathematics classroom. I proceed to argue for the use of problems that require to students to seek out and evaluate information outside of the classroom as a way of providing an opportunity for the exercise of rational dependence. This is followed by an illustrative example drawn from a qualitative study in which the author observed the implementation of such problems. The example illustrates the perils of encouraging a critical stance towards quantitative claims without encouraging students to seek out the expertise of relevant epistemic communities as part of their analysis.

Teachers of mathematics are increasingly confronted with students who have access to varied sources of information outside of the classroom. The premise of a word problem can be challenged by a quick glance at Wikipedia, calculations that were formerly impossible to carry out on a graphing calculator can now be performed with ease using an answer engine like Wolfram Alpha, and students can track down discussions of test problems on various Q&A web-sites. Teachers have the option of either ignoring this reality, treating it as an imminent threat to be guarded against, or encouraging students to engage with *information-based problems* (Walraven, Brand-Gruwel, & Boshuizen, 2008) or those problems that require "students to identify information needs, locate corresponding information sources, extract and organize relevant information from each source, and synthesize information from a variety of sources" (Walraven et al., 2008, p.623). In this paper, I argue that the use of information-based problems is not just preferable, but that it is also necessary if mathematics instruction is to help prepare students for the quantitative arguments that they may expect to encounter in their everyday lives (Paulos, 1988; Steen, 2001). Following this argument, I will describe an example drawn from my dissertation research that exemplifies one of the problems that can occur when students critique a quantitative argument without attending to the nature of the epistemic community that the argument stems from.

Students have always been able to draw on information outside of the classroom if they so choose, but a convergence of demands bound up with three different conceptions of literacy are serving to push this issue to the forefront. (1) A move to attend to how reading and writing is conducted in the disciplines and to use this as a way of thinking about how to teach those disciplines means that mathematics teachers must think more carefully about how those who work in STEM fields locate and evaluate mathematical resources (Schleppegrell, 2007; Moje, 2007; Shanahan & Shanahan, 2012). (2) While mathematics instruction has long been charged with an instrumental role with respect to training in the STEM fields, there have been more recent moves to develop curriculum that works toward a mathematical proficiency that can serve any individual in their everyday lives. This is sometimes framed as quantitative literacy (Steen, 2001), mathematical literacy, quantitative reasoning or granted a more limited scope as a type of statistical literacy (Cullinane & Treisman, 2010; Watson, 2013) and has seen some realization in courses offered by many colleges for non-STEM majors who need to fulfill a mathematics requirement as part of their liberal arts education. (3) Researchers in the information sciences have long recognized the importance of instruction in order to facilitate college and high school students' ability to research topics at the library (Rader, 2002).

Information-Based Problems in the Math Classroom

What does all of this mean for mathematics instruction? While the importance of information-based problems for disciplinary literacy is easy to justify as long as one accepts that information-seeking is an important part of practice in the disciplines, it requires a little more unpacking to explain why this type of instruction might have a place in mathematics instruction specifically. As a start, we can imagine an applied mathematics problem where students are given an editorial in which the author argues that federal guidelines on fuel efficiency will end up costing the country more money than it will save (Diefenderfer, 2009). Students are asked to read thid editorial and then provided with several guiding questions that encourage the students to analyze the numerical argument contained in the article. While this activity is a legitimate applied mathematics problem, the real-world context (see Figure 1) suggests other directions that such a problem could be taken.

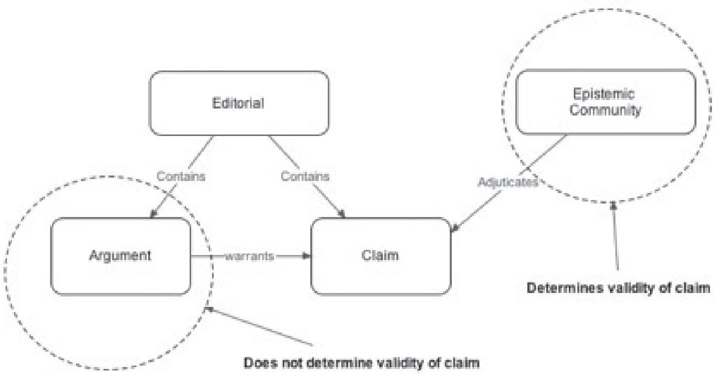

Figure 1: Relationship between Editorial, Claim, and Epistemic Community

For example, if a reader were to actually want to determine whether the editorial's claim was true or not, they would want to locate the relevant *epistemic community* (Haas, 1992), or that community that possesses the expertise to tentatively rule on the truth of the claim. The problem as originally stated does not afford the student an opportunity to seek out and evaluate those sources of information that might either corroborate or challenge the argument found in the editorial. An information-based problem, on the other hand, with its requirement that the student seek out and evaluate sources outside the classroom, would give the student the opportunity to challenge and/or corroborate the premises of the argument presented in the editorial. It may turn out that numbers were misrepresented by the author or that straw-men arguments were used to mischaracterize the opposition. It could even be the case, as it often is in the inevitable compression characteristic of newspaper articles, that an invalid argument was presented in support of a position that was nonetheless held to be true by the relevant epistemic community for other legitimate reasons. None of these aspects of the situation could be discovered without recourse to information sources outside of the article itself.

Why Rational Dependence?

Why should information-based problems be introduced to a quantitative literacy course? Looking at the situation from the other way around, one can ask a different question: How much can we learn about highly contextualized problems that are mathematical in full or in part without relying on pertinent experts and expert communities? The epistemologist John Hardwig (1985) argues that we are very limited in this respect with his notion of *epistemic dependence:*

> The layman's appeal to the intellectual authority of the expert, his epistemic dependence on the expert, and his intellectual inferiority to the expert (in matters on which the expert is expert) are all expressed by the formula […]: [The layman] has good reasons to believe [the expert] has good reasons to believe that p. But the layman's epistemic inferiority and dependence can be even more radical—in many such cases, extensive training and special competence may be necessary before [the layman] could conduct the necessary inquiry. And, lacking this training and competence, [the layman] may not even be able to understand [the expert]'s reasons, or, even if he does understand them, he may not be able to appreciate why they are good reasons (Hardwig, 1985, p. 338)

I would only supplement this description with three points. First, while the layperson/expert dichotomy may sound divisive, it should be remembered that we are all in the same boat. That is to say, an expert is only an expert in a highly circumscribed domain and the moment she steps beyond that limited sphere her status reverts to that of a layperson. Thus, we do not live in a world that consists of two classes: experts and laypeople. Rather, we are all laypeople who are occasionally granted the opportunity (or responsibility) to contribute our thinking on those few matters that touch upon the areas of our expertise. Second, while Hardwig (1985) refers to individual experts, he also acknowledges that anybody who might be called an expert also belongs to and depends on a community that enables, shares, and affirms the expert's status as an expert. Any scholar must trust in the findings of other scholars whether their object of study is particle physics or Milton. Thus, it may make more sense to speak of an epistemic community (Haas, 1992) with respect to a given topic rather than isolated experts.

Finally and crucially, as Harvey Siegel (1988) noted, an individual's awareness of her epistemic dependence does not give her leave to relinquish her rationality, it only highlights that she must use her reason to decide between sources of information rather than relying on her direct knowledge of the field. In his paper responding to Hardwig (1985), Siegel spelled out many of the considerations that would occupy a hypothetical layperson, Smith, prior to accepting the opinion of a hypothetical expert, Jones, on a topic of import:

> She must, first, determine whether this is a question about which she ought to defer to expert opinion: Does she have the ability to become expert and reason about the matter directly? If so, is it rational for her to expend the requisite time and effort? Supposing that one of these questions is answered in the negative (rationally so answered, of course), Smith must ask herself: Is this question one concerning which there is likely to be reliable expert opinion? Is it likely to admit of evidence which an expert might come to recognize? Is the relevant field of inquiry sufficiently developed and sophisticated that there is reason to regard its practitioners as experts? And what about Jones: What sort of expert is she? Would other experts agree with her? Is she rightly regarded as an expert? Does she have a conflict of interest in this case? Is there any other special circumstances which renders her expert judgment in this case problematic? (Siegel, 1988, p. 4)

By way of contrast with the educational goal of intellectual independence (Norris, 1997), I hereafter refer to skills displayed by Siegel's hypothetical student as *rational dependence*, or the ability of an individual to rationally seek out and draw from relevant epistemic communities. The situation outlined is a confrontation between a layperson and an expert but we can nuance that by recalling what was stated above about claims being warranted by epistemic communities (Adler & Haas, 1992; Collins & Evans, 2008). It is also worth noting that Kuhn's (1991) investigation of personal epistemologies suggested that it would be misleading to talk about trust in an absolute way. Kuhn found that college students tend to move from a position of absolute trust in experts to a position of radical relativity where no one's perspective is privileged and finally to a point of view that acknowledges that there exist good reasons to at least provisionally privilege the opinions of some individuals over others when it comes to subjects that fall within their area of expertise. These considerations do nothing to mitigate the essential problem: people necessarily come by much of their knowledge of the world around them through trust in a variety of epistemic communities rather than through direct investigation. In my dissertation research, I investigated the potential for introducing this type of activity into the classroom through the introduction of information-based problems. While my dissertation presents a more thoroughgoing cross-case analysis of what I found, I will present here a single illustrative anecdote in order to better illustrate some of the perils of encouraging a critical stance in one's students without fostering a concomitant sense of rational dependence.

The Vaccine–Autism debate

Ivan[1] is a mathematics teacher who was teaching a topics in mathematics course for non-STEM majors at Delta University, a regional research university. The majority of his students were juniors and seniors fulfilling their mathematics requirement prior to graduation. I collaborated with Ivan to develop a couple of information-based problems that he introduced to his class, the first of which required his students to choose a controversial topic and to analyzing the sampling strategy used by some relevant polls and research articles -- the students had just begun a unit on statistics and experimental design. Ivan's primary goal for this information-based problems was to, in his own words, instill a "critical sense" (Ivan, 4/17/14, Line 150) in his students. He described this as "something that should immediately click that says 'something fishy going on here'" (Ivan, 4/17/14, Line 149) when a student encounters a mathematical claim that does not make sense. I offer up this specific episode without making any claims about typicality, as there was only one time that anything like this occurred during the two information-based problems. Rather, it serves to illustrate a potential outcome of an assignment in which students are encouraged to examine published research with a critical eye.

Ivan began this activity by demonstrating what he wanted students to do by displaying an article on the board and discussing where it met and fell short of a proper experimental design. As it happened, the first presenting group adhered exactly to this format on a superficial level. The students described the article that they would go on to critique:

> Taylor: We ran across this new article and it's this one up here, the project by the CDC, government funded, and the claim of it is that there is no causal relationship between vaccines and autism rates in children. With this being said—
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> Michael: Only 3 of the 8 managed care organisations were chosen for the study. A thousand, roughly, children participated. 256 had autism and 752 did not. And which I thought the eligibility for being selected was that you had to be born January 1st, 1994 to December 31st and you had to be previously enrolled in one of the NCOs from birth until your seventh birthday. Currently enrolled at the same time of sample selection, and you had to live within 60 miles of the study. You had to be between 6 and 13 years old at the time the study was selected and had to live with your biological mother since birth and, lastly, you had to speak good English. And you were excluded if you had any of the links to autism. (First Session, First Presentation, 3/3/14, Lines 22–33)

After spending a little more time describing the different sections of the article, Taylor criticized the choice of sample size by saying,

> It has 1008 participants, and I don't know about you guys but I feel like that's pretty low for a sample size. You're trying to

> sample a huge autism outbreak and you want to find out what causation or correlation is, you're not just going to sample a thousand from a 60 mile radius. You're going to try to get a nice randomized sample. It never says in any of the articles how it was randomized. It never—it just says that they were picked from the 1008 and it says they were picked from a 60 mile radius, so you wonder how they got these people. It never explained in the actual research article. (Taylor, First Session, First Presentation, 3/3/14, Lines 45–51).

According to Taylor, the problem with the sample size is that it "feels [...] pretty low" given the claim that was being made. This concern could be a result of the common misconception that a larger population requires a larger sample size or that the sample size should be thought of as a fixed proportion of the population (Huck, 2009). The student's subsequent concern about how "it never says in any of the articles how it was randomized" could be an honest concern although it is not clear what would count as sufficient evidence of proper randomization.

Immediately following her critique of the sampling method that Taylor used this evidence to level an accusation at those who funded the study:

> And it never said about the 256 people that did contract autism from these vaccines, they never talked about, even though they did, why it wasn't a cause. Also that kind of led to a sample bias because it's such a small sample size and, I think, well what we thought, was that maybe the government could be trying to cover up something because they didn't want us to think that they're vaccines which are regulated through the government, they wanted to make everything seem okay, but this study just seemed a little bit fishy. With all the, 'you can't be in the study if you're linked to autism', you can't be in the study if you don't live within 60 miles, and only taking a thousand and some people, I think that - Definitely a larger sample size would be more accurate and not have bias and it was very problematic to put so many restrictions on who could be in the study or not. (Taylor, First Session, First Presentation, 3/3/14, Lines 51–61).

While "sample bias" was one of the concepts that had been covered in the lectures that led up to these presentations, these students were not using it in the proper sense. While they said "that kind of led to sample bias", the referent of "that" is not apparent. It may be that the speaker is suggesting that the various restrictions related to recruitment resulted in a sample that was not actually random and that it was instead designed to be biased in order to exclude people who had been diagnosed with autism and whose inclusion would reveal the "true" link between autism and vaccination.

During my interview with Ivan after the class session, he expressed great frustration at how that particular presentation had played out. While the students had satisfied the terms of the assignment with respect to the tasks that they were asked to engage in (i.e., describe and critique the sampling methodology of a study), they had misapplied the statistical concepts that they were being taught. They had critiqued the sample size without actually having grounds to do so and complained about the restrictions placed on the sample without giving a reason for why it would be problematic. Ivan stated that he was frustrated because the students did not draw on what had been covered in the previous lectures when they criticized the paper. Further, while he did not say so explicitly in class, there is evidence that he disagreed with the conclusion that the students were arguing. During the questioning period, he contested a claim that was made about the chemical thimerosal being used as an additive in vaccines by pointing out that thimerosal had not been added to vaccines in the last ten years. This is notable because it is the only time that he called into question a point of fact that was not mathematical in nature. He told me after the class that he was aware that the students had "an agenda" (Ivan, 3/3/14) but he did not know how to intervene.

While, as noted above, there was a mathematical critique to be made of the students' presentation, it is notable that these students followed the directions of the assignment and that they engaged with the assignment to a degree that other teachers with whom I worked, and even Ivan in other circumstances, would have found creditable. The problem was not the students' engagement or enthusiasm, rather Ivan appeared to be upset because the students were failing to acknowledge the recognized consensus of the relevant epistemic community (Plotkin, Gerber, & Offit, 2009), i.e. those involved in epidemiological research. I submit both that Ivan's frustration is an understandable byproduct of an attempt to create a more contextualized mathematical problem without encouraging students to look beyond the articles at hand in order to assess the state of the debate writ large and that there exist other activity structures that could give students the motive along with the opportunity to engage in just that type of investigation. For example, I describe elsewhere (Erickson, forthcoming) a classroom in which students were given a very similar problem to the one described above, but in this case they were required to discuss the credibility of the sources that they found with their peers and to deliver a verdict on the most credible of those sources. Some of these students also chose the vaccine-autism debate but the subsequent conversations touched on the controversy surrounding Andrew Wakefield's retracted paper for *The Lancet* (Wakefield et al., 1998) as well as the understandable fears of parents confronting with the media's interpretation of the controversy. I do not make any claims here about the relative efficacy of the two different activity structures for encouraging rational dependence in classrooms other than those that I observed first-hand. Rather, I present these examples in order to (1) demonstrate that the reasoned dependence on the expertise of others is an important component of a productive critical attitude and to (2) show that teachers with similar goals and supports may nonetheless provide very different opportunities to carry out the exercise of rational dependence based on how information-based problems are operationalized in the classroom.

Conclusion

As long as mathematics instruction is directed not just towards the preparation of future STEM-professionals but also intended to empower students to engage with the mathematics that they will encounter in their everyday lives, I argue that students must both be encouraged and supported in the development of rational dependence. There is nothing to be served by allowing students to think that they can, or should, critically evaluate issues on their own if only they had the necessary background knowledge. This is both an unreasonably high bar that can serve to exclude rather than include people in public debate, and it misrepresents the collectively evolving nature of scientific knowledge itself. And there is everything to be gained by helping students realize that they depend on others and providing them with tools to help them navigate the cacophony of voices that they will encounter in the real world. Finally, I want to stress that an acknowledgment of our individual limitations ought to be seen as empowering. When it is allowed that we can (and should) cede certain judgements to expert communities, we are broadening our world in an essential way. This attitude empowers individuals to make more decisions of consequence and brings them into contact with the greatest of human accomplishments in all their plurality.

Notes

1. All proper names of people or places are pseudonyms.

References

Adler, E., & Haas, P. M. (1992). Conclusion: epistemic communities, world order, and the creation of a reflective research program. *International Organization, 46*(1), 367–390.

Collins, H., & Evans, R. (2008). *Rethinking expertise.* Chicago: University of Chicago Press.

Cullinane, J., & Treisman, P. U. (2010). *Improving developmental mathematics education in community colleges: A prospectus and early progress report on the Statway Initiative.* National Center for Postsecondary Research Working Paper.

Diefenderfer, C. L. (2009). *Case studies for quantitative reasoning: A casebook of media articles*. Pearson Custom Publishing.

Erickson, A. (2015, forthcoming). *Counting on the knowledge of others: Rational dependence in the mathematics classroom.* (Unpublished doctoral dissertation). University of Michigan, Ann Arbor.

Haas, P. M. (1992). Banning chlorofluorocarbons: epistemic community efforts to protect stratospheric ozone. *International Organization, 46*(1), 187–224.

Hardwig, J. (1985). Epistemic dependence. *The Journal of Philosophy, 82*(7), 335–349.

Huck, S. W. (2009). *Statistical misconceptions*. New York, NY: Routledge.

Kuhn, D. (1991). *The skills of argument.* Cambridge: Cambridge University Press.

Moje, E. B. (2007). Developing socially just subject-matter instruction: A review of the literature on disciplinary literacy teaching. *Review of research in education, 31*(1), 1–44.

Norris, S. P. (1997). Intellectual independence for nonscientists and other content-transcendent goals of science education. *Science Education, 81*(2), 239–258.

Paulos, J. A. (1988). *Innumeracy: Mathematical illiteracy and its consequences.* London: Viking Penguin.

Plotkin, S., Gerber, J. S., & Offit, P. A. (2009). Vaccines and autism: a tale of shifting hypotheses. *Clinical Infectious Diseases, 48*(4), 456–461.

Rader, H. B. (2002). Information literacy 1973-2002: A selected literature review. *Library trends, 51*(2), 242–259.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly, 23*(2), 139–159.

Shanahan, T., & Shanahan, C. (2012). What is disciplinary literacy and why does it matter?. *Topics in Language Disorders, 32*(1), 7–18.

Siegel, H. (1988). Rationality and epistemic dependence. *Educational Philosophy and Theory, 20*(1), 1–6.

Steen, L. A. (2001). *Mathematics and democracy: The case for quantitative literacy.* The National Council of Education and the Disciplines.

Wakefield, A. J., Murch, S. H., Anthony, A., Linnell, J., Casson, D. M., Malik, M., ... & Walker-Smith, J. A. (1998). RETRACTED: Ileal-lymphoid-nodular hyperplasia, non-specific colitis, and pervasive developmental disorder in children. *The Lancet, 351*(9103), 637–641.

Walraven, A., Brand-Gruwel, S., & Boshuizen, H. (2008). Information problem solving: A review of problems students encounter and instructional solutions. *Computers in Human Behavior, 24*(3), 623–648.

Watson, J. M. (2013). *Statistical literacy at school: Growth and goals.* New York, NY: Routledge.