Social Class and the Visual in Mathematics

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This is a discussion and theoretical paper arguing for a social class analysis of the use of the visual in mathematics teaching. Whilst each topic has a considerable literature, they are rarely linked in the mathematics education literature. However I argue for an inextricable connection between non-textual modes of thinking and communication and the SES gap in mathematical achievement.

Introduction

Each academic year, the prospect of yet again teaching fractions to a class of low achieving challenging adolescents strikes abject frustration in mathematics teachers throughout the world. Yet the reality is many young people fail to understand even basic mathematics. How we get to this position where, after 9-10 years of compulsory schooling, we are still trying to convince some children that 1/4 = 2/8 is nothing short of an international scandal. Worryingly, this is after decades of curriculum reviews, policy changes and millions spent on research. One feature is that achievement is not equitably spread throughout society; children from less affluent homes do disproportionately worse than those bought up in relative affluence. This after all is the *raison d'etre* of MES. Much research has attempted to articulate this relationship, whilst much more has ignored it, through denial, or the misguided belief that by supporting the affluent all will benefit through the *"trickle down"* principle. There is one feature of school mathematics that is perhaps more common than most: a dependence on *language* and textual communication to the near exclusion of other modes of communication—most notably the *visual*. But whilst *"language is a marvelous tool for communication, but it is greatly overrated as a tool for thought"* (Reed, 2010, p. 1).

The importance of how children mentally represent mathematics is widely acknowledged. Mental representations are ways of constructing mental models, and the process of teaching is to support that construction. To argue, think and reason *visually* is something not yet at the forefront of teachers' conceptions of mathematics learning. Hence the use of diagrams and visuals may currently play a very superficial, insignificant or minor role. Some go further suggesting teachers discourage visual and diagrammatic forms as less valid than the symbolic (Morgan, 2004). Others take an alternative stance arguing that visual thinking is epistemologically central to mathematics (Giaquinto, 2007); others offer a neurological justification:

> Visualisation extends working memory by using the massively parallel architecture of the visual system to make an external representation function as an effective part of working memory. (Crapo, Waisel, Wallace, & Willemain, 2000, p. 220)

It is well known that we receive, hold and process information in various ways - for example through an auditory stimulus, or a visual stimulus and that these channels combine in a non-trivial way. However, there still appears to be a lack of attention given to the construction and manipulation of mental models especially those drawing on visual-spatial processes. Dawe puts as an imperative for teachers to *"consciously link visual images, verbal propositions and memories of activities, involving the manipulation of physical objects"* (Dawe, 1993). Whilst visualisation and mental imagery are cognitive processes evident from birth, there would appear to be different levels of individual facility yet little evidence of explicit instruction at school. These skills are open to enhancement through classroom activity—e.g. with evidence of gains in higher education after explicit training in visualisation (Lord, 1985, 1990). The need to be proficient in visualisation is important in many fields such as engineering, medicine, construction—in fact it may be difficult to find a field of employment where it is not. The lack of direct instruction in visual facility in school mathematics is therefore worrying.

Visual skills useful in learning and doing mathematics might include: constructing a mental image or mental model of a mathematical artifact or process, developing and using a mental representation, constructing representative diagrams, describing (representing) images and models, mental rotation. Here there are further claims of gender and class differences in spatial skills (Linn & Peterson, 1985) and age effects (Bishop, 1978). Linn and Peterson argue there is evidence of males using a holistic approach, with females taking an analytical approach. Bishop argues the there is a developmental process moving from topological, through 2d to increased sophistication in 3d. The widespread use of computer gaming by young people may also be enhancing their visual and spatial ability. However as with all developmental processes, environment and social factors play a significant part.

Those pupils who do well at mathematics tend to be able to use a range of strategies that help them develop the capacity to engage at a number of levels within a range of topic areas. Mathematical thinking engages us in a wide range of cognitive and social practices and so defies a clear well bounded definition - one learns to use strategies, to see relationships and structures in a wide variety of forms. Furthermore in participating in these practices, we engage in logico-deductive thinking, use language to express ideas, talk to communicate and convince but in addition we also use a spatial, visual mode of thinking which helps us see, experience and master various mathematical objects and relationships.

The Importance of Economics

However all this ignores the fact that children do not start school —or life—on an equal footing. It is well known in an extensive literature that there is a significant difference in the levels of achievement of children from different social backgrounds (or social classes), and these early differences expand during the course of compulsory schooling (Alexander et al., 1988; Geary, 1994, 2006). For most pupils, visualization is not instinctive but *"one learns to 'see'"* (Whiteley, 2000, p. 4) though we might observe in many mathematics classroom pupils learning to *repeat* or learning to *say*. Children from disadvantaged backgrounds have forms of knowledge that do not allow them to fit so well into the expectations of schools as do those from more affluent or middle-class homes. Whilst this seems to be true generally, there seems to be specific differences in learning of mathematics (Case, Griffin, & Kelly, 1999) where the most significant and consistent predictor of academic achievement in school seems to be the parental income, which has an effect stronger even than parental educational background. Where ethnicity and gender are factors, they are usually confounded with SES (Jordan, Huttenlocher, & Levine, 1992, p. 652); in the first two years of formal education, school makes little difference to this (Stipek & Ryan, 1997, p. 721). Yet, one major impediment to the amelioration of mathematical teaching and learning around the world is that much work in mathematics education is so politically focused as to ignore the social class basis of mathematics learning. Whilst this is lamentable, it is not surprising; indeed it would be surprising if the field of mathematics education were quarantined from the left-right/radical-conservative dispositions that exist everywhere else.

Crucial to understanding the influence of class on learning is specifying the types of mathematical knowledge on which the discrepancy is present (Siegler & Ramani, 2009). On nonverbal numerical tasks, preschoolers' performance does not vary significantly with economic background (Ginsburg & Russell, 1981; Jordan et al., 1992; Jordan, Levine, & Huttenlocher, 1994). However, on tasks with verbally stated or written numerals, the knowledge of preschoolers and kindergartners from low-income families lags far behind that of peers from more affluent families. The differences are seen on a wide range of tasks: recognizing written numerals, reciting the counting string, counting sets of objects, counting up or down from a given number other than 1, adding and subtracting, and comparing numerical magnitudes (See Ginsburg & Russell, 1981; Griffin, Case, & Siegler, 1994; Jordan et al., 1992; Jordan, Kaplan, Olah, & Locuniak, 2006; Siegler & Ramani, 2009; Starkey, Klein, & Wakeley, 2004; Stipek & Ryan, 1997).

Significant is the argument that the problem lies deep within the way in which schools divorce children from the informal intuitive forms of understanding they had experienced before formalized education. Ginsburg and Russell (1981) investigated the associations of social class and race with early mathematical thinking arguing that early mathematical thought develops in a robust fashion regardless of social class and race and that school failure, specifically in mathematics cannot be explained by initial cognitive deficits (p. 56) a finding in conflict with many early years teachers' beliefs. However, Ginsburg and Russell (1981) argue that it was cognitive *competence* not a cognitive *deficiency* that might be in existence. Specifically, low-income children seemed to have a less developed set of what Case and Griffin call "*central conceptual structures*" (1990a) that went on to underpin future cognitive development specifically of mathematical and numerical processes (Griffin et al., 1994, p. 36), that without these detailed structures early on, children would go on to develop a "rote" approach to learning which would limit the scope of their level of achievement (p. 47). However, through a taught "Rightstart" programme focusing on conceptual bridging, multiple representation and affective engagement, Griffin et al. (1994) were able to demonstrate elimination of differences. The importance of looking at the competencies of children very early on is the more significant neurological influence of the developing brain, since

> children's early mathematical capacities show a considerable degree of differentiation by social class during the years when the neurological circuitry on which they depend is showing its most rapid development (Case et al., 1999, p. 148)

Case et al. go on to argue that whilst SES differences are not observable at birth they do begin to appear around 3 years old (Ginsburg & Russell, 1981), but by kindergarten this had become a year and a half difference in capabilities (Case et al., 1999, p. 131). These early differences in mathematical knowledge have lasting effect as preschoolers' performance on tests of mathematics is predictive of mathematical achievement at age 8, 10 and 14 and even in later in upper secondary school (Duncan et al., 2007; Stevenson & Newman, 1986). This stability of individual differences in mathematical knowledge reflects to some extent the usual positive relationship between early and later knowledge, but the stability of individual differences in mathematics is unusually great. This might be because mathematics is something of a secret garden, avoided by low SES parents (Siegler & Ramani, 2009):

> Observations of homes and preschools, as well as the self-reports of teachers and parents, suggest that the home and preschool environments provide children with relatively little experience where their attention is focused on mathematics, far less than literacy-oriented experience (Siegler & Ramani, 2009, p. 558)

However, there is some evidence, that social class effects upon the development of mathematical skills is more marked for verbal than non-verbal forms (Jordan et al., 1992). Children from middle-income families do better when the mode of representation is verbal. Yet where the mode of representation is visual or nonverbal, the social class gap is much reduced possibly because verbal and written forms of communication are less prioritized in working class families. Alternatively for working class families *"knowledge that has been constructed directly from their own actions on objects as well as their observations of the world"* applies equally to development of visual and nonverbal modes (Jordan et al., 1992, p. 651). Siegler and Ramani (2009) take this need for privileging of the non-verbal further but argue that whilst pre-school children from more affluent backgrounds perform better on some numerical tasks than disadvantaged children, this differential performance can be partially alleviated by regular playing of linear board games - consistent with the hypothesis that playing board games contributes to differences in numerical knowledge among children from different backgrounds, children from middle-income families reported playing far more board games (though fewer video games) than their low-income peers, indicating part of the gap between low-income and middle-income children's mathematical knowledge when they enter school is due to differing play experiences (Ramani & Siegler, 2008; Siegler & Ramani, 2009, p. 557). Given that these same disadvantaged children report playing board games at home with much less frequency than the affluent children, Siegler conjectured that this might be partially influential in not providing the cognitive experience that would move them forward (see also Dehaene, 2011):

> …board games provide a physical realization of the mental number line, hypothesized to be the central conceptual structure for understanding numerical operations in general and numerical magnitudes in particular" (Siegler & Ramani, 2009, p. 546)

Allocation of blame to working-class parents is common amongst politicians and some researchers, yet interviews with parents in low-income families indicate that many believe the primary responsibility for teaching mathematics lies with the professionals in schools (Holloway, Rambaud, Fuller, & Eggers-Pkirola, 1995; Tudge & Doucet, 2004) a perhaps not surprising position given the self-importance with which the teaching profession surrounds itself. Indeed Tudge and Doucet (2004) studied children's exposure to explicitly mathematical activities in their own homes, other people's homes, and child care centres, supporting this assertion.

> A majority of children from working class backgrounds were observed engaging in mathematical play or mathematical lessons in 0 of 180 observations. If it is indeed correct that working-class parents look to preschool settings to provide children with mathematics experiences . . . our data suggest that they are mistaken—we found no evidence that children are more likely to be engaged in mathematical activities . . . in formal childcare centers than at home (Tudge & Doucet, 2004, p. 36).

The role of schools and preschools then might usefully be expanded by building stronger and more explicit links between school and home activities that support mathematical thinking and visualisation.

Conclusions

For many, the claim that economically disadvantaged children do less well at school will be hardly controversial, or new. Yet the next stage of that argument often escapes some. This is the *"so what"* question. A damaging stance is to take a deficit perspective, that *"these children"* need remediation, that they miss out of stimulation in the home, that both children and parents *"lack aspiration"*, and even worse, they need a more practical curriculum for a practical future, focusing on *"the basics"* reinforced though repetition. In a study of 262 US preschool children, Stipek and Ryan (1997) argue that economically disadvantaged preschool children very quickly developed a more negative view of their own competencies and negative attitudes to school, both which lead to a decline in motivation leading to potential future depression of achievement (p. 722).

> Disadvantaged children are every bit as eager to learn as their more economically advantaged peers. They do however have much further to go in terms of their intellectual skills and, as schools are presently organized, they do not catch up. (Stipek & Ryan, 1997, p. 722)

In a society—and school system—that extols only the virtues of the rich, famous and successful, this is perhaps quite iniquitous but not surprising. Their suggested alternative is to develop instructional methods that will decrease the gap in cognitive competencies specifically targeting the self-esteem and interest of disadvantaged children. This is not an easy policy to enforce, especially since narrowing the gap acts against the social and economic interests of those who benefit from being at the head of the gap. Many studies have indicated ways in which parents might support children in seeing and thinking more mathematically, yet the practices being advanced might be more readily seen in middle class families: taking advantage of opportunities to practice spatial thinking (Joh, Jaswal, & Keen, 2011; Newcombe, 2010; Pruden, Levine, & Huttenlocher, 2011); playing construction games that challenge children to recreate a design from a sample or design (Ferrara, Golinkoff, Hirsh-Pasek, Lam, & Newcombe, 2011), encouraging children to gesture when they think about spatial problems (Cook & Goldin-Meadow, 2006; Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001), playing with tangrams and jigsaw puzzles (Levine, Ratliff, Huttenlocher, & Cannon, 2012), creating and explaining maps (Kastens & Liben, 2007), practicing mental rotation skills including through computer games (Terlecki, Newcombe, & Little, 2008; Wright, Thompson, Ganis, Newcombe, & Kosslyn, 2008). Studies of early cognition do suggest potentially useful strategies which might benefit learners of mathematics from more disadvantaged backgrounds. Whilst much work on the links between disadvantage and achievement look to generic social structures, a look toward studies of cognitive development point toward more specific aspects of how that process is operationalized. As a consequence, there is sufficient evidence to consider a greater examination of the mode of communication and representation as playing a significant role.

Diagrams can be important in grasping mathematical concepts and solving mathematical problems but diagrams are only one of the ways we come to "see" things. If learners are going to keep learning new ideas and new structures in mathematics they have to be able to imagine objects that they can't actually see, but also need to be able to visualize all sorts of processes and objects and interpret visual information. However, interpreting diagrams and helping pupils visualize is rarely explicitly taught in school mathematics lessons and the result is that pupils see and experience school mathematics as written, largely formulaic, with symbols that need to be textually manipulated.

There is another element to this. It is also well known that young people from disadvantaged backgrounds find it harder to succeed at school mathematics than those young people who have experienced relative economic privilege. Schools don't make this any easier for them by placing all such pupils together in the same mathematics groups and restricting their curriculum and linguistic opportunities, but also restricting the development of alternative forms of representation. Research has consistently shown that young people specifically from low SES backgrounds do less well (Noble, Norman, & Farah, 2005), and on spatial tasks the SES difference is confounded with gender (Levine et al., 2005). This may be due to their experiences as young children, the toys they have (or don't have!) the use they make of maps etc. Hence there is a need to explore the use of visualization in teaching and learning mathematics and how teachers and pupils can be supported to develop imagery and mental manipulation as a natural part of mathematics—which after all gets increasingly abstract the further you go.

This forces us to ask, what use is made of visualisation in teaching mathematics and do groups at different levels get the same experience, particularly those in lower-attaining groups populated by pupils with low SES backgrounds? Low teacher expectations can influence the methods that teachers use and can limit the access of these groups to higher order skills. Children from low SES backgrounds may well have an impoverished mathematical experience before school but their progress may be restricted further if teaching methods do not allow them access to the appropriate opportunities for development. If visualisation is potentially beneficial to mathematical development then how and when is this taught in schools, but more importantly, how might it reduce the SES gap in achievement?

The use of visual methods in early education does require some understanding and interpretation of simple visual representations, such as gesture, and these have a social context. In using these representations teachers make assumptions that may put certain social or cultural groups at a disadvantage. Pupils may hear the same words and see the same gestures but the resulting message may not be the same. If maths students from Papua New Guinea failed to interpret certain features of mathematical diagrams in a 'conventional' way because of their cultural experience (Lean & Clements, 1981) then socio-cultural differences may well affect students' in all our schools. Their perceptions of visual representations may well be very different to those expected by their teacher making it difficult for them to then grasp the associated mathematical concepts. The difference between individual mental images and those held by the teacher lies in the variation in interpretation of the shared external representation, which, itself depends on the image generated by memory and their existing underlying conceptual understanding. A visually rich prior experience and a socio-cultural background that embraces the norms of the school culture may be influential in the early development of visualisation for individuals in schools.

In conclusion, there is a need to examine two key issues in mathematics which are too often kept apart by looking at the spatial and visual capabilities of pupils from low SES or less affluent backgrounds. We probably can't do much about improving their social and economic backgrounds; we might however be able to do something about enhancing some of the key skills which they have not previously been required to focus on—visual acuity, visual reasoning, and mental representations.

References

Alexander, K. L., & Entwisle, D. R., Blyth, D. A., & McAdoo, H. P. (1988). Achievement in the first 2 years of school: Patterns and processes. *Monographs of the Society for Research in Child Development, 53*(2, Serial No. 157).

Bishop, J (1978). Developing students' spatial ability. *Science Teacher, 45*(8), 20–23.

Case, R., & Griffin, S. (1990a). Child cognitive development: The role of central conceptual structures in the development of scientific and social thought. In C.-A. Hauert (Ed.), *Developmental psychology: Cognitive, perceptual-motor, and psychological perspectives*. North-Holland: Elsevier.

Case, R., Griffin, S., & Kelly, W. (1999). Socioeconomic gradients in mathematical ability and their responsiveness to intervention during early childhood. In D. Keating & C. Hertzman (Eds.), *Developmental health and the wealth of nations: Social, biological, and educational dynamics* (pp. 125–149). New York, NY: Guilford Press.

Cook, S., & Goldin-Meadow, S. (2006). The role of gesture in learning: Do children use their hands to change their minds? *Journal of Cognition and Development, 7*(2), 211–232.

Crapo, A., Waisel, L., Wallace, W., & Willemain, T. (2000). Visualization and the process of modelling: A cognitive-theoretic view. *Proceedings of the Sixth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*. Boston, Massachusetts.

Dawe, L. (1993). Visual imagery and communication in the mathematics classroom. In M. Stevens, A. Waywood, D. Clarke & J. Izard (Eds.), *Communicating mathematics: Perspectives from classroom practice and research* (pp. 60–76). Hawthorne, Victoria: Australian Council for Educational Research.

Dehaene, S. (2011). *The number sense: how the mind creates mathematics*. Oxford: Oxford University Press.

Duncan, G., Dowsett, C., Claessens, A., Magnuson, K., Huston, A., Klebanov, P., . . . Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*, 1428–1446. doi: 10.1037/0012-1649.43.6.1428

Ferrara, K., Golinkoff, R., Hirsh-Pasek, K., Lam, W., & Newcombe, N. (2011). Block talk: Spatial language during block play. *Mind,* *Brain, and Education, 5*(3), 143–151.

Geary, D. (1994). *Children's mathematics development: Research and practical applications.* Washington, DC: American Psychological Association.

Geary, D. (2006). Development of mathematical understanding. In D. Kuhn & R. Siegler (Eds.), *Handbook of child psychology: Volume 2. Cognition, perception, and language* (6th ed.) (pp. 777–810). Hoboken, NJ: Wiley.

Giaquinto, M. (2007). *Visual thinking in mathematics: An epistemological study*. Oxford: Oxford University Press.

Ginsburg, H., & Russell, R. (1981). Social class and racial influences in early mathematical thinking. *Monographs of the Society for Research in Child Development, 46*(6, Serial No. 193), 1–69.

Goldin-Meadow, S., Nusbaum, H., Kelly, S., & Wagner, S. (2001). Explaining math: Gesturing lightens the load. *Psychological Science, 112*(6), 516–522.

Griffin, S., Case, R., & Siegler, R. (1994). Rightstart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at risk for school failure. In K. McGilly (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 25–49). Cambridge, MA: MIT Press.

Holloway, S., Rambaud, M., Fuller, B., & Eggers-Pkirola, C. (1995). What is "appropriate practice" at home and in child care? Low-income mothers' views on preparing children for school. *Early Childhood Research Quarterly, 10,* 451–473.

Joh, A., Jaswal, V., & Keen, R. (2011). *Imagining a way out of the gravity bias: preschoolers can visualize the solution to a spatial problem. Child Development, 82*(3), 744–750.

Jordan, N., Huttenlocher, J., & Levine, S. (1992). Differential calculation abilities in young children from middle- and low-income families. *Developmental Psychology, 28*(4), 644–653.

Jordan, N., Kaplan, D., Olah, L., & Locuniak, M. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, 77*(1), 153–175.

Jordan, N., Levine, S., & Huttenlocher, J. (1994). Development of calculation abilities in middle- and low-income children after formal instruction in school. *Journal of Applied Developmental Psychology, 15*, 223–240.

Kastens, K., & Liben, L. (2007). Eliciting self-explanations improves children's performance on a field-based map task. *Cognition and Instruction, 25*(1), 45–74.

Lean, G., & Clements, K. (1981). Spatial ability, visual imagery, and mathematical performance. *Educational Studies in Mathematics, 12*(3), 267–299.

Levine, S., Ratliff, K., Huttenlocher, J., & Cannon, J. (2012). Early puzzle play: A predictor of preschoolers' spatial transformation skill. *Developmental Psychology, 48*(2), 530–542. doi: 10.1037/a0025913

Levine, S., Vasilyeva, M., Lourenco, S., Newcombe, N., & Huttenlocker, J. (2005). Socioeconomic status modifies the sex difference in spatial skill. *Psychological Science, 16*(11), 841–845.

Linn, M., & Peterson, A. (1985). Emergence and characterisation of gender differences in spatial ability. *Child Development, 56*, 1479–1498.

Lord, T. (1985). Enhancing the visual-spatial aptitude of students. *Journal of Research in Science Teaching, 22*, 395–405.

Lord, T. (1990). Enhancing learning in the life sciences through spatial perception. *Innovative Higher Education, 15*(1), 5–16.

Morgan, C. (2004). What is the role of diagrams in communication of mathematical activity? In B. Allen & S. Johnston-Wilder (Eds.), *Mathematics education: Exploring the culture of learning* (pp. 134-145). London: RoutledgeFalmer.

Newcombe, N. (2010, Summer). Picture This. *American Educator*, 29–43.

Noble, K., Norman, F., & Farah, M. (2005). *Neurocognitive correlates of socioeconomic status in kindergarten children.* Developmental Science, 8, 74–87.

Pruden, S., Levine, S., & Huttenlocher, J. (2011). Children's spatial thinking: does talk about the spatial world matter? *Developmental Science, 14*(6), 1417–1430.

Ramani, G., & Siegler, R. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*(2), 375–394.

Reed, S. (2010). *Thinking visually.* Hove: Psychology Press.

Siegler, R., & Ramani, G. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology, 101*(3), 545–560.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly, 19*, 99–120.

Stevenson, H., & Newman, R. (1986). Long-term prediction of achievement and attitudes in mathematics and reading. *Child Development, 57*, 646–659.

Stipek, D., & Ryan, R. (1997). Economically disadvantaged pre-schoolers: Ready to learn but further to go. *Developmental Psychology, 33*(4), 711–723.

Terlecki, M., Newcombe, N., & Little, M. (2008). Durable and generalized effects of spatial experience on mental rotation: Gender differences in growth patterns. *Applied Cognitive Psychology, 22*, 996–1013.

Tudge, J., & Doucet, F. (2004). Early mathematical experiences: Observing young Black and White children's everyday activities. *Early Childhood Research Quarterly, 19*, 21–39.

Whiteley, W. (2000). Dynamic geometry programs and the practice of geometry. http://www.math.yorku.ca/Who/Faculty/Whiteley/Dynamic.pdf

Wright, R., Thompson, W., Ganis, G., Newcombe, N., & Kosslyn, S. (2008). Training generalized spatial skills. *Psychonomic Bulletin & Review, 15*(4), 763–771. doi: 10.3758/PBR.15.4.763