History of Mathematics Textbooks and the Construction of Mathematical Subjectivity

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In this paper, I consider how our ideas about mathematics shape our individual and cultural relationship to the field. Specifically, I argue that histories of mathematics can play a role in the construction of a mathematical subjectivity that is both gendered and racialized. I examine a best-selling history of math textbook which uses a biographical approach to tell the story of the mathematician-hero. I show how this approach to the history of mathematics results in the construction of a mathematical subjectivity that limits who can see themselves within it. My paper ends with a consideration of a textbook that constructs a much more expansive mathematical subjectivity by teaching readers how to engage in the process of creating their own accounts of the historical development of mathematics.

One of the key ways we come to understand mathematics is via the histories that we tell of its development as a field of knowledge. There are many who argue that the history of mathematics should be included in standard mathematics curricula (Calinger, 1996; Katz, 2000). At the very least, many mathematics textbooks have small sidebars that give brief histories of famous problems and biographies of the mathematicians associated with them. In this paper, I argue that the way we tell the history of mathematics often results in the construction of a mathematical subjectivity that is normatively masculine and white. David Stinson (2013) calls this normative mathematical subjectivity the "white male math myth," and demonstrates the negative impact it can have on African American mathematics students. In order to show how this normative mathematical subjectivity is constructed in history of mathematics textbooks, I examine the work of David Burton (2010), whose *The History of Mathematics: An Introduction* is the best-selling history of mathematics textbook in the

US (Smoryński, 2008). I use Michel Foucault's theory of the author function to analyze Burton's account of one of the most well-studied periods in the history of Western mathematics, Newton's discovery of the calculus.

Foucault's (1998) central argument in his essay, "What is an Author?" is that the author is a subject position produced by discourse and that the figure of the author works to regulate the proliferation of meaning by limiting who is allowed to speak and what is allowed to be said. He begins by arguing that, "The coming into being of the notion of "author" constitutes the privileged moment of individualization in the history of ideas, knowledge, literature, philosophy, and the sciences" (p. 205). Enlightenment notions of the self—essential, self-contained, autonomous, rational—were exemplified by the idea of the author. The image of the author is represented by what Foucault calls the fundamental category of "the-man-and-his-work"—the author sits in a room of his own, creating his next literary masterpiece (p. 205). Foucault argues that the author is a discursive construction that allows us to maintain the illusion that we are autonomous selves in control of our own destinies. Histories of mathematics like Burton's perpetuate this illusion, constructing a figure of the mathematical author that epitomizes the author function. Consider the following description of Newton's work from Burton's (2010) textbook:

> While Newton was forced to live in seclusion at home [due to the plague], he began to lay the foundations for his future accomplishments in those fields with which his name is associated—pure mathematics, optics, and astronomy. During these two "golden years" at Woolsthorpe, Newton made three discoveries, each of which by itself would have made him an outstanding figure in the history of modern science (p. 391).

Just in this short passage, key elements of the Enlightenment construction of the author appear. The idea of the "the-man-and-his-work" is central to the story of Newton's discoveries and Burton establishes the image of the mathematical author, sitting in a room of his own with the phrase "forced to live in seclusion." A component of this trope is the idea that history does not act on the subject, rather the subject acts upon history; he makes history. This is apparent in the reversal we see in the above passage. Newton might have been forced by circumstances to live in seclusion, but he utilized that time to do work that "made him an outstanding figure in the history of modern science."

With this phrase, a Western ideal is constructed—a rational, productive individual, whose work not only garners him a place in history, but effectively makes history. This is a common trope in biographical history writing. Jean-Michel Raynaud (1981), in his essay, "What's What in Biography," notes:

> All biographies that you can read deal with the same story of which the hero is not one particular individual, but the Individual as such manifested as being the powerful agent acting on everything, on groups, on events, on history. Biography is therefore the story which reveals the Individual, the essential myth of our European society (p. 93).

By embedding biographical information into the history of mathematics, a mathematical author gets constructed in the same heroic way that the subjects of most biographies are constructed—as capital-I Individuals, whose life and work serve to change the course of history. This is similar to the construction of the author that Foucault (1998) critiques. As both the source and origin of a text, the author stands outside the text; he is beyond it. This corresponds with the biographical construction of the individual—the powerful agent acting upon history and, as such, standing outside of history. Foucault identifies how problematic this construction is, arguing that the concept of the author functions to regulate who can see themselves within it. He uses literary texts, specifically focusing on narratives, to analyze how the author function operates via discourse, but he acknowledges in numerous places throughout his essay that a variety of texts work to construct an author.

By analyzing the ways in which a mathematical author is constructed in history of mathematics textbooks, we can begin to understand how a normative mathematical subjectivity gets created. According to Foucault (1998), "the manner in which [discourses] are articulated according to social relationships can be more readily understood…in the activity of the author function" (p. 220). There is a connection between the social relationships that determine our understanding of who can occupy the position of author and the discursive construction of the author figure. Foucault goes on to argue that the subjectivity that is constructed is a privileged subjectivity, clearly stating that "one could also, beginning with analyses of this type, re-examine the privileges of the subject" (p. 220). He suggests a set of questions that get at the relationship between the author function and the construction of subjectivity:

> How, under what conditions, and in what forms can something like a subject appear in the order of discourse? What place can it occupy in each type of discourse, what functions can it assume, and by obeying what rules? … What are the modes of existence of this discourse? Where has it been used, how can it circulate, and who can appropriate it for himself? *What are the places in it where there is room for possible subjects? Who can assume these various subject functions?* [emphasis added] (Foucault, 1998, pp. 221-222).

Analyzing the ways in with the figure of the author is discursively constructed gives us insight into how subjectivity itself is discursively constructed. If we can identify the ways the mathematical author function limits who can "appropriate it" for him or herself, then we can gain insight into who "can assume these various subject functions."

So when thinking about the figure of the author, we need to move beyond the simple attribution of a text or a mathematical theorem to a specific individual. Rather, according to Foucault (1998), the idea of the author results from various cultural constructions, in which we choose certain attributes of an individual as "authorial" attributes, and dismiss others. What attributes signify the mathematical author? Suzanne Damarin (2000) argues that there are two conflicting discourses that work to construct the figure of the mathematically able in Western society: a discourse of power and a discourse of deviance. The power associated with mathematical ability is both cultural and economic. In the highly technological society we live in, mathematical achievement can translate into economic success in the form of job skills that lead to success on the job market. But even more so, mathematical achievement brings with it cultural capital in corporate, political and academic circles. Heather Mendick (2006), in her study of the discursive construction of mathematics found that mathematics is variously framed as, "a route to economic and personal power within advanced capitalism," "a source of personal power," and "the ultimate form of rational thought and so a proof of intelligence" (p. 18). Yet there is associated with mathematical achievement a degree of deviance as well. Because mathematics is understood to be the ultimate form of rational thought, those who engage with mathematical knowledge are often thought of as removed from normal human occupations and leisure. So much so, that the trope of the mentally ill, yet brilliant, mathematician, is quite common, whether we are discussing the "beautiful mind" of John Nash or the dangerous insanity of Unabomber Ted Kaczynski (Damarin, 2000). Damarin describes the major factors that mark the mathematically able: "brilliant but remote from reality, different from 'the rest of us', and bearing bodily marks, to wit, 'it's in the genes'" (p. 77).

This description of the mathematically able corresponds with our cultural understanding of mathematics itself. Mathematical knowledge production is conceived to be an individual cognitive activity, where the mathematician, working in isolation, discovers a mathematical theorem using logical, rule-based reasoning to develop and modify the work of those who came before him. According to Foucault (1998), "these aspects of an individual which we designate as making him an author are only a projection, in more or less psychologizing terms, of the operations that we force texts to undergo, the connections that we make, the traits that we establish as pertinent, the continuities that we recognize or the exclusions that we practice" (p. 213). We can see this projection happening in histories of mathematics that embed biographical information within the story of the historical development of mathematical knowledge. Our cultural understanding of the mathematician is intimately connected to our understanding of mathematics itself. According to Paul Ernest (1992), we have in our culture a popular image of mathematics as "difficult, cold, abstract, ultra-rational, and largely masculine" (n.p.). I argue that in Burton's (2010) history of mathematics textbook, a mathematical subjectivity is constructed, via the figure of the mathematical author, that reflects this popular image of mathematics.

We can see elements of this image throughout Burton's textbook—the ideas of deviance and power; the construction of mathematical work as difficult, cold and abstract; the celebration of rationality above all else; and the characterization of rationality as masculine. Burton certainly refers to the various ways in which Newton is constructed as deviant and I will show this in just a moment. But far more prominent than any description of Newton's deviance are descriptions of his influence, the power that he wielded and his status in seventeenth-century England. Burton's characterization of Newton follows that of most histories of mathematics. It is simply understood that Newton is a hero—a key figure in the history of the West; his mathematical work shaped Western history for all time. Consider the hyperbole in Burton's (2010) introductory paragraph to the section on Newton:

> It remained for a still greater mind, Isaac Newton, to give the scholarly world the synthesis for which it yearned. Newton's *Philosophiae Naturalis Principia Mathematica* (1687) was the climax of the soaring intellectual thought that marked the seventeenth century, the Century of Genius. Probably the most momentous scientific treatise ever printed, it aimed, in Newton's words, "to subject the phenomena of Nature to the laws of mathematics" (p. 386).

The cultural capital that Newton gains from his mathematical achievements becomes clear in Burton's account. After the publication of his *Principia*, Newton receives a royal appointment as warden, then master, of the British mint. He becomes the president of the Royal Society in 1703 and is re-elected to the role, without opposition, until his death twenty-four years later. In 1705, Queen Anne knighted Newton, a farmer's son and the first scientist to be so honored. Yet alongside this cultural capital, there is ample evidence that Newton suffered from mental illness, which many attribute to the strain of his mathematical genius. According to Burton (2010),

> The severe mental exertion of composing the *Principia* took its toll. Newton began to suffer from insomnia and lack of appetite, and by 1692 his mental health had deteriorated to the point where he was afflicted with some sort of nervous illness (p. 406).

The difficulty of genius-level mathematical thinking is established. The coldness of such work is clear in Burton's constant references to Newton's obsession with his work, his desire to work alone, his propensity to see colleagues as enemies, and his lack of a family. The abstract nature of Newton's work is well-established. Burton frequently refers to the fact that few of Newton's contemporaries could understand his work. Newton's lectures during his time at Cambridge University as the Lucasian Chair of Mathematics were so rigorous and "severely mathematical" that they were largely unattended (Burton, 2010, p. 392).

Finally, the characterization of mathematics as masculine becomes clear in Burton's (2010) description of the lack of women in Newton's own life—his mother abandons him as a young boy, he never marries—and in Burton's attempt to discuss the only two female mathematicians associated with Newton: Maria Agnesi and Émilie du Châtelet (pp. 430-432). Burton does not incorporate biographical information about these women into the sections on the history of the calculus. Rather, he creates a separate section for them at the end of the chapter. He briefly describes their peripheral role in the discovery of the calculus: Agnesi wrote one of the first textbooks on the calculus and du Châtelet translated Newton's *Principia* into French. While their work is certainly considered important vis-à-vis the work of Newton, nowhere does Burton describe them with the aplomb he reserves for the heroic Newton. In fact he ends his section on du Châtelet as follows: "It may be said of Émilie du Châtelet that she was more an interpreter of the accomplishments of others than a creator of original science" (Burton, 2010, p. 432).

Burton's biographical writing about Newton solidifies the construction of mathematics and mathematical subjectivity as difficult, cold, abstract, and ultra-rational, but also as largely masculine. This construction of mathematical subjectivity serves a purpose, according to Paul Ernest (1992). If mathematics is understood in this way:

> … then it offers access most easily to those who feel a sense of ownership of mathematics, of the associated values of western culture and of the educational system in general. These will tend to be males, to be middle class, and to be white. Thus the argument runs that the popular image of mathematics described above sustains the privileges of the groups mentioned by favouring their entry, or rather by holding back their complement sets, into higher education and professional occupations, especially where the sciences and technology are involved (n.p.).

History of mathematics textbooks contribute to this phenomenon, constructing a normative mathematical subjectivity that limits who can identify themselves within it, thus limiting who can access mathematics itself. In the last part of this paper, I examine a history of mathematics textbook that challenges this construction of mathematical subjectivity.

Luke Hodgkin, in his (2005) *A History of Mathematics From Mesopotamia to Modernity*, argues that most textbooks present the history of mathematics as a series of facts and that within those textbooks "the live field of doubt and debate which is research in the history of mathematics finds itself translated into a dead landscape of certainties" (p. 4). Hodgkin very clearly states in his preface that his intention in the writing of this textbook is to engage students, not just in the history of mathematics, but in the making of historical narratives. In this way, Hodgkin approaches the history of mathematics very differently than does Burton. Burton (2010) presents the history of mathematics as an factual narrative of intellectual development in Western culture. He chooses to include biographical elements in this narrative in order to engage the interest of the reader, arguing that "there is no sphere in which individuals count for more than the intellectual life" (p. x). According to Burton, because mathematicians "stand out as living figures and representatives of their day, it is necessary to pause from time to time to consider the social and cultural framework in which they lived" (Burton, 2010, p. x). The construction of the normative mathematician-hero is a result of the choices Burton makes in his textbook. In contrast, Hodgkin introduces his textbook with a discussion of the contingent and constructed nature of scholarship in the history of mathematics, differentiating for his readers between the absolutist histories that are normally presented in textbooks and professional research in the field. Hodgkin

> … hope[s]to introduce students to the history, or histories of mathematics as constructions which we make to explain the texts that we have, and to relate them to our own ideas. Such constructions are often controversial, and always provisional; but that is the nature of history (Hodgkin, 2005, Preface, para. 2).

To this end, Hodgkin begins each chapter by outlining the field of historical literature, both primary and secondary sources. He then asks readers to evaluate and interpret this literature on their own—to engage in the act of history-writing.

This powerful approach to the teaching of the history of mathematics acknowledges what Michel-Rolph Trouillot (1995) argues is a key element in the study and creation of historical knowledge: figuring out how history works by studying how it is produced. Trouillot articulates two sides of historicity—what actually happened and the narrative of what happened—and argues that a focus on the process of producing history is the only way to "uncover the ways in which the two sides of historicity intertwine in a particular context" (Trouillot, 1995, p. 25). This very much corresponds with Hodgkin's approach in his history of mathematics textbooks. He explicitly states in his introduction that "the emphasis falls sometimes on history itself, and sometimes on *historiography*: the study of what historians are doing" (Hodgkin, 2005, p. 4). By inviting readers into the process of producing history, Hodgkin both exposes the pluralistic and contradictory nature of historical knowledge and invites readers to generate their own interpretations. By encouraging his readers to create history—to work with primary and secondary texts, to create their own stories about the mathematicians and mathematical knowledge they are studying—he is empowering readers to actively engage with the knowledge, both historical and mathematical. In so doing he positions his readers to ask some of the questions with which Foucault (1998) ends his essay on the author function: What are the places in [this discourse] where there is room for possible subjects? Who can assume these various subject functions?

Hodgkin's (2005) approach in his history of mathematics textbook results in a more pluralist construction of mathematical subjectivity, allowing more people to see themselves within it. Because of how closely intertwined mathematical knowledge is with histories of its development, producing historical narratives about mathematics necessarily involves working with mathematical knowledge. By inviting readers into the process of constructing historical narratives, Hodgkin makes room for possible subjects in ways that Burton's textbook does not; he opens up the possibility of assuming mathematical subjectivity. Trouillot (1995) argues very clearly that the process of producing history influences the construction of subjectivity; writing about what happened empowers the writer as a subject. I consider two ways that Hodgkin expands possibilities with regard to the construction of mathematical subjectivity. First, I look at the way he challenges the trope of the mathematician/hero by portraying figures like Isaac Newton with all of their human foibles intact. Though he might have invented the calculus, Newton was not a very likable character, nor was he the ideal hero that Burton's text makes him out to be. I then look at the way Hodgkin invites readers into the assessment and interpretation of historical evidence and mathematical argument. In this way, Hodgkin takes historical and mathematical knowledge that is characterized by Burton as monumental in scope, and, instead, asks that his readers reinterpret those historical narratives and work with the mathematical knowledge in order to critically engage with it.

Perhaps the most striking part of Hodgkin's (2005) chapter on the calculus is that he characterizes what Burton (2010) calls, "the climax of the soaring intellectual thought that marked the seventeenth century, the Century of Genius" (p. 386) and "the most momentous scientific treatise ever printed" (p. 386) as follows:

> Any mediocre person can break the laws of logic, and many do. What Newton and Leibniz did was to formalize the breakage as a workable system of calculation which both of them quickly came to see was immensely powerful, even if they were not entirely clear about what they meant (Hodgkin, 2005, p. 162).

Hodgkin is referring here to the use of infinitesimals, those infinitely small quantities that are continuously vanishing as you work a calculus problem. By characterizing the invention of the calculus in this way, Hodgkin humanizes it and makes the work that Newton and Leibniz did relatable. In a similar vein, Hodgkin calls Newton and Leibniz's work an invention, asking "so what was it that Newton, and later Leibniz, invented?" (p. 169). Calculus becomes not a Platonic truth discovered by extraordinary genius, but the work of humans who were not quite sure what it was that they were toiling away at, only that it seemed to provide a new tool for solving some age-old problems.

Hodgkin not only characterizes this moment in the history of mathematics in a very different way than do most historians of mathematics, he also uses the exercises in his chapter to ask the reader to engage with the mathematical knowledge as it was presented within the original texts, not just as passive problem solvers, but as active assessors of that knowledge. For example, he asks readers to consider a passage from one of Newton's papers and then assess the strengths and weakness of the mathematical argument within that passage (Hodgkin, 2005, pp. 171-2). In a similar fashion, Hodgkin asks readers to engage in the production of historical knowledge, as well. He provides an excerpt from one of Leibniz's papers on the calculus and asks readers to give "a historical take" on the paper by posing the following questions: 1) what was Leibniz trying to communicate?; and, 2) how might this communication have been received by a reader? (p. 173). Hodgkin then discusses how a reader might answer these questions, teaching the reader how to work through Leibniz's original text. Not only is he enabling his reader to engage with the original texts, he is also providing them with the historiographical tools to assess that text and the impact it might have had at the time of publication. This allows readers to put the invention of the calculus into context, not as the most monumental discovery of human history, but as an innovative, but highly confusing and contentious, invention.

This approach to the history of mathematics makes room for readers to understand themselves as part of the process of producing knowledge about mathematics and the history of mathematics; they enter into subjectivity as they read through and engage with Hodgkin's text. Hodgkin's textbook powerfully challenges normative constructions of mathematical subjectivity by challenging the trope of the mathematician-hero constructed in most history of mathematics textbooks and by expanding who can understand themselves as engaging with mathematical knowledge. It is important, as we move forward, that we continue to challenge those history of mathematics texts that perpetuate a very limited cultural construction of mathematical subjectivity and that we encourage the publication of more histories of mathematics that invite readers to engage directly with that history and with mathematical knowledge itself. In doing so, we will construct a much more expansive mathematical subjectivity that may allow those from marginalized groups to understand themselves as mathematical subjects.

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