Discussing Mathematical Modeling Course in a Long Distance Course

Daniel Clark Orey
Universidade Federal de Ouro Preto, Brazil
oreydcead@gmail.com

Abstract: The research related to critical and reflective dimensions of mathematical modeling is seeking identity, definition, and objectives. As well, it is developing a sense of its own nature and potential for research methods used in order to legitimize pedagogical action. It is necessary to discuss the importance of philosophical and theoretical perspectives found in critically reflective dimensions of mathematical modeling. As well, the importance of a virtual learning environment that helps students to develop critical-reflective efficiencies has become increasingly important to enable the exploration of theories related to critical mathematical modeling, distance interactions, and transactional distance by using long-distance technologies. These interactions are triggered by lessons placed on platforms, which are virtual learning environments (VLE) and that enable the use a combination of technology, teaching and the learning of specific content. By developing discussion forums and videoconferences, professors and tutors are able to analyze interactions enabled by these tools, which contributed to the reflective development of the elaboration of mathematical models in the VLE.

Key words: Critical and Reflective Dimensions, Mathematical Modeling, Long Distance Education

Introduction

In recent years, Brazil has experienced an accelerated social and economic growth with many accompanying social changes. The country is the 8th largest economy in the world, sponsored the 2014 World Cup and it is sponsoring the Olympics in Rio de Janeiro in 2016. Brazil has undergone a process of modernization in relation to infrastructure, including that of health and education.

Nationally, a process of developing teacher competencies and the training of new teachers is making a difference in school and community quality. The most expedient, economical, indeed reasonable method to do this is by integrating the long distance education and accompanying multimedia technologies (Alves, 2011). To increase access to a wider audience, the use of Moodle as the platform and freeware is used; which has enabled the *Universidade Aberta do Brasil* (UAB) system to the democratize and increase access to higher education.

Because of the social changes resulting from contemporary scientific and technological development in Brazil, the study of diverse curricular and methodological proposals has become vital. The need to update and upgrade professional development for all teachers has raised discussions in relation to new institutional methods and resources in order to meet the demand for specialized teacher education programs; this is even more relevant in the context of mathematics education. Long distance learning in Brazil offers teacher education programs to prospective teachers in places and in diverse contexts that have historically suffered limited access to higher education.

In a long distance modality, instruction is performed by using a variety of technologies as well as special organizational and administrative arrangements (Moore & Kearsley, 2007). Several actions conducted by the Brazilian Federal Government were developed for teaching and learning in long distance modalities. The Ministry of Education's plan has been to invest in distance learning and create a new digital era for informational and communicational technologies in order to support teaching practices, initial and continued education, and professional development (Brasil, 2005).

According to Brazilian law, *Long Distance Education* is characterized as "an educational modality in which didactical and pedagogical mediation is facilitated by the teaching and learning processes that occur with the use of a variety of informational and communicational technologies". In this process, students and teachers develop educational activities in diverse and distinct localities and periods (Brasil, 2005). It is necessary to work beyond the concepts of this law in order to understand the curricular, didactical, and pedagogical action plan for this educational modality, which has direct influence on the quality of education offered to diverse students and communities throughout Brazil. In 2003, there were 50,000 Brazilian students enrolled in 52 long distance courses and, one decade later, in 2013, there were 1.2 million students enrolled in 1.258 courses in Brazil.

It is important to emphasize the relevance in the preparation of long distance courses where it is necessary to know the learning needs of a very large number of diverse groups of students and their unique conditions in which they live. However, it is not enough to merely enhance access to this kind of education without changing and adapting processes and methods of teaching and learning regarding available technological resources. Therefore, in order to establish processes and methods for developing long distance learning mathematics courses, it is more than necessary to implement courses such as modeling in this educational modality.

It has become necessary to establish a system of long distance learning processes and methods based on existing theories regarding these research fields. In so doing the authors have explored and applied this in a *Seminar in Mathematical Modeling* in a long distance mathematics undergraduate course in Brazil.

This course is offered entirely in a distance environment, and is mediated through technological tools on the internet. The *Centro de Educação Aberta e a Distância* (CEAD) at the *Universidade Federal de Ouro Preto* (UFOP) has come to integrate instruction, technology, content and pedagogical methods in order to reach a diversity of students. Long distance students represent approximately, 21,4% of the 14,000 UFOP students in three states: Bahia, Minas Gerais, and São Paulo.

Figure 1: States of São Paulo, Minas Gerais, and Bahia in Brazil

At the time of this writing, in just this university alone, there are approximately 3000 students enrolled in 4 (four) undergraduate majors such as *Mathematics*, *Geography*, *Pedagogy*, and *Public Administration*. These students are enrolled in 5 (five) Graduate Courses such as *Sustainable Schools*, *Pedagogical Coordination*, *School Management*, *Media & Education*, and *Pedagogical Practices*.

Long distance students access courseware and instruction through 35 *polos*, which are long distance learning centers equipped with computer labs, internet, libraries, and tutorial assistance. In many localities the UAB educational centers (polos) has become the lone access to the Internet and library resources. UFOP is one of the oldest public institutions of higher education in Brazil and provides one of the largest distance education programs in the country.

Long Distance Learning

Worldwide, distance education has grown quickly. Beginning initially with the use of mail-order courses, it transitioned quickly to include radio and television. Once associated with mailed printed materials, it has now facilitated the dissemination and democratization of access and has now moved to incorporate the internet and *MOOCS*. It has become a key element in the democratization for many, many countries and now allows access to education and professional development opportunities once only given to elite members of society.

In Brazil, as mentioned earlier, it has allowed a portion of the population that traditionally has had difficulty in accessing public education, due to a variety of geographical or economic reasons, to advance. The basic idea of distance education is very simple: students and teachers are in different locations during all or most of the time in which they either learn or teach (Moore & Kearsley, 2007).

Although this type of education might, in some ways, hinder traditional teacher-student relationships, it also allows students who had never had access to professors or teachers to gain contact. Distance education technologies answer a critical need for those who deserve initial and/or continuing education opportunities. Distance education allows for educators and learners to break barriers related to time and space, and allows for interactivity and information dissemination. Distance education environments are open systems that are composed of "flexible mechanisms for participation and decentralization, with control rules discussed by the community and decisions taken by interdisciplinary groups" (Moraes, 1997, p. 68).

This approach also allows interactions between teachers who prepare instructional materials and strategies, with tutors, who, in the case of Brazil, provide hands-on face-to-face assistance at the polos. In Brazil, tutors are tasked give encouragement, to assist students in their activities and tasks, guide them in organization, helping them learn to use search tools, libraries, and offer help in basic skills (most notably in writing and mathematics).

These interactions are triggered by lessons in *Virtual Learning Environments* (VLE) and enables the teaching and learning of specific content to a wider audience. These features have enabled the development of a large variety of educational methodologies that utilize web interaction channels and aim to provide needed support in the achievement of VLE curricular activities.

The Process of Critical and Reflective Dimensions of Mathematical Modeling

According to the Brazilian National Curriculum for Mathematics (Brasil, 1998), students need to develop their ability to solve problems, make decisions, work collaboratively, and to communicate effectively using mathematics. This approach helps our students to face challenges posed by society by turning them into flexible, adaptive, reflective, critical, and creative citizens.

This perspective is related to the sociocultural dimensions of mathematics, which are closely associated with ethnomathematics (D'Ambrosio, 1990). This aspect emphasizes the role of mathematics in society by highlighting the importance of analyzing the critical and reflective dimensions of mathematical models in order to help individuals to solve everyday challenges present in the contemporary society (Rosa & Orey, 2013).

This context allows mathematical modeling to provide both real and concrete opportunities for students to discuss the role of mathematics as well as the nature of mathematical models (Shiraman & Kaiser, 2006). It also could be understood as a language to study, understand, and comprehend problems faced by a specific community (Bassanezzi, 2002). In this process, the purpose of mathematical modeling is to develop students' critical and reflective skills that enable them to analyze and interpret data, to formulate and test hypotheses, and to develop and verify the effectiveness of the mathematical models. In so doing, the reflection on reality becomes a transformative action, which seeks to reduce the degree of complexity of reality through the choice of systems that it represents. This approach originates the critical and reflective mathematical modeling cycle, which allow students to act in order to transform society (Rosa & Orey, 2015).

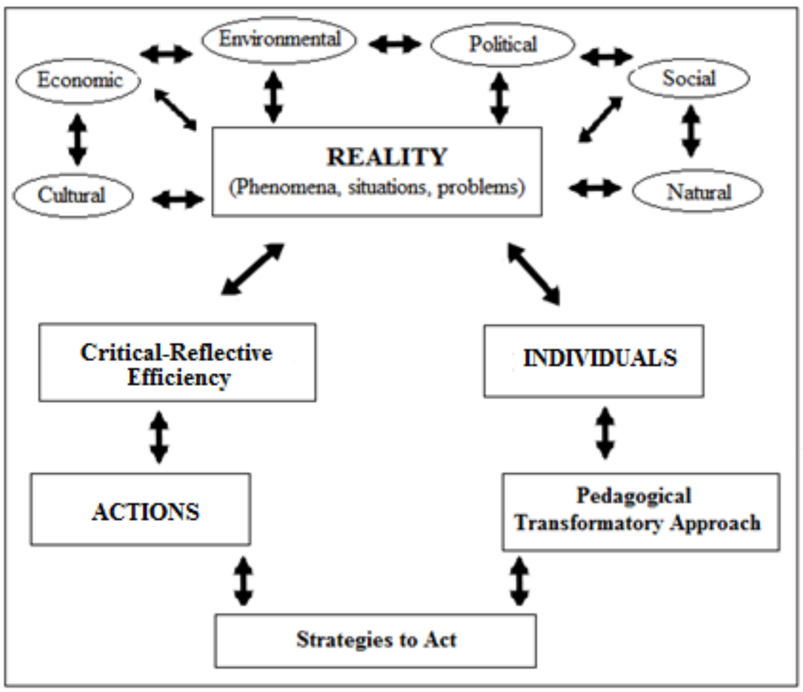


Figure 2: Critical and Reflective mathematical modeling cycle

Systems taken from a students' reality allow them to learn to make representations of this reality by developing strategies that enable them to explain, understand, manage, analyze, and reflect on all parts of this system. This process aims to optimize pedagogical conditions for teaching so that students are better able to understand a particular phenomenon in order to act effectively transform it according to the needs the community. The application of critically and reflective dimensions of mathematical modeling makes mathematics a dynamic and humanized subject. This process fosters abstraction, the creation of new mathematical tools, and the formulation of new concepts and theories.

Thus, an effective way to introduce students to mathematical modeling in order to lead them towards the understanding of its critical and reflective dimensions is to expose them to a wide variety of questions, problems and themes. As part of this process, questionings about the themes used to explain or make predictions about phenomena under study through the elaboration of mathematical models that represent these situations (Rosa & Orey, 2013).

Because models are understood as approximations of reality, the elaboration of models does not mean that it develops a set of variables that offer qualitative representations or quantitative analysis of the system. In this direction, to model is a process that checks whether the parameters are critically selected for the solution of models in accordance to the interrelationship of selected variables from holistic contexts of reality. It is not possible to explain, know, understand, manage, and cope with reality outside the holistic contexts (D'Ambrosio, 1990).

This aspect of traditional learning prevents students' access to creativity, conceptual elaboration, and the development logical, reflective, and critical thinking. According to this perspective, any dimension of mathematical modeling facilitates the development of competencies, skills, and abilities that are necessary, indeed vital for students to play a transformative role in society (Rosa & Orey, 2007).

Therefore, critical and reflective mathematical modeling may be considered a learning environment in which students inquire and investigate problems that come from reality. In this environment, students work with real problems and use mathematics as a language for understanding, simplifying, exploring, and solving situations in an interdisciplinary fashion (Bassanezi, 2002). In other words, critical mathematical modeling is a method using applied mathematics that was transposed to the field of teaching and learning as one of the ways to use and connect reality in the mathematics curriculum (Barbosa, 2006).

In the context of critical and reflective mathematical modeling processes, students communicate by using hermeneutics (written, verbal, and non-verbal communication) to verify if social actions and norms are modified by communication, which can be developed through the virtual learning environments. It is in this kind of knowledge that meaning and interpretation of communicative patterns interact to construct and elaborate the community understanding that serves to outline the legal agreement for the social performance. In this learning environment, students control and manipulate technological tool, which is gained through empirical investigations and governed by technical rules in the VLE.

In this mathematical modeling process, students apply this instrumental action when they observe the attributes of specific phenomena, verify if a specific outcome can be produced and reproduced, and know how to use rules to select different and efficient variables to manipulate and elaborate mathematical models.

Critical Mathematical Modeling Process in the Virtual Learning Environments

In literacy and language learning environments, learners very quickly learn to communicate through oral and or written forms of language. Early on, learners see the importance of written narratives, prose, and poetry that allow them to quickly see the beauty and power of language, and to incorporate that beauty into their lives. Contrast this to the learning of mathematics, where in mathematics classes, learners are often subjected to endless rote memorization of algorithms and grammar and pages of exercises, often without context to the learners' lives, experiences, values or communities. Because of the race to cover material for testing, learners are rarely given the opportunity to see direct connections to what they are learning and practice how they actually use mathematics in their own contexts and reality.

Mathematics is often referred to as a language. However, mathematics has become a language that is taught without giving leaners the opportunity to really communicate its use! That is, learners spend years learning the grammar, but do not write even rudimentary forms of mathematical prose or poetry (models). To this author it is truly a sad state of affairs. It is not until learners reach advanced mathematics that the few that survive this cruel process, are afforded bot the luxury and opportunity to engage in communicating and creating new ideas using the beauty and power found in the language of mathematics.

For this writer, most people come to detest mathematics because to them it is stuck in endless disconnected and truly boring rules and drills in the use of mechanical mathematical grammar without them ever being able to write or communicate in this beautiful, elegant, synthetic but powerful language (Rosa, 2010).

To those of us who have been privileged to understand the beauty and elegance of mathematics, this is deeply sad. In many places in Brazil, a strong culture of inquiry has developed in the mathematics education community by using critical mathematical modeling, and is influenced by the philosophies and work of both Paulo Freire (2005) and Ubiratan D'Ambrosio (1986). In preparation for rigorous university entry exams, Brazilian students are encouraged to reflect upon, engage in, debate, and dialogue mathematically to resolve problems they find in their own contexts, neighborhoods and environments. These opportunities often use modeling and ethnomathematics and become the first opportunity that mathematics learners have to write a mathematical poem (Rosa & Orey, 2011).

For example, data gleaned from a study about transportation conducted by Orey and Rosa (2014) in 2013 in a course offered to mathematics majors in mathematical modeling showed that students acquired information through interviews with citizens and public transport users in their respective towns. In this regard, questions related to the situations presented in the interviews served as starting point to the elaboration of mathematical models.

In June 2013, early in a Seminar on Mathematical Modeling, the country erupted in mass demonstrations against the growing problem of corruption and over spending in relation to preparation for the 2014 World Cup tournament. Just in our small college town of Ouro Preto, 10,000 people marched from the university campus to the main square of the city. What sparked this national mass movement was a sudden spike in transportation fares in urban transportation systems. For those who do not use mass transit something as minor (20-cent rise) created a very difficult problem for who live in the large metropolis of São Paulo, Rio de Janeiro, Belo Horizonte, Salvador, Fortaleza, and Brasília.

Some long daily commutes became R$50 (about U$14) roundtrips five or six times a week and for many finally became untenable. Normally a week or so is devoted to bringing consensus with students and generating a number of themes, and to make use of this particular historic circumstance the instructor consulted with the tutors and students and together we agreed that transportation would be the theme. Eight polos were participating in the seminar. The instructor asked the tutors at each polo to organize the students into smaller working groups of 4 or 5 students. Over a period of 5 weeks, students were led through the steps, and groups were required to post evidence of their work on line.

Synchronous virtual classes were held. Critical and reflective mathematical modeling lessons were transmitted through video conference sessions. Lessons were organized and activities and projects were posted on the Moodle Platform. Discussion forums were also developed in order to prepare students for the modeling process. By the end of a 16-week course, there were 4 synchronous/virtual meetings in which the development of the mathematical models of each group of students was discussed. The course calendar that contained the description of the course, the terms of the proposed activities, and the dates and times of synchronous was published in the VLE. Approximately, every two weeks there were activities and questions to be worked on by the students and sent to the tutors and the professor through specific links in the Moodle Platform.

Pedagogical and didactic strategies were used to promote the interactions with the students and professors and tutors in order to contribute to the process of teaching and learning critical and reflective mathematical modeling. The process becomes more like coaching, than traditional teaching. The resources used for this purpose were the discussion forums and videoconference. Through these tools, it was possible to promote dialogues between all participants in our VLE. In addition to promoting interactions, the professor took care in the preparation of teaching materials, such as the structure and policies of the activities available on the VLE. Due to the many perceived needs of the students during this course, the professor created supplemental materials and short video-lessons in order to lead students gradually into the modeling process, so they were able to improve their performance in carrying out the modeling proposed activities.

It is important to highlight here the design that was applied in the use of digital communication technologies in the development of this course. These technologies included:

a) Videoconferences that enabled the integration of students, tutors and the professor for socialization and clarification of questionings; which allowed for a collaborative environment for sharing experiences on the proposed themes and promoted students attendance in the polos to develop their modeling projects. The use of videoconferences proved to be effective because it has sufficient teaching resources for conducting synchronous classes. In this perspective, knowledge is translated in a dialogical way so these technological tools can be used as instruments to help students to critically think about problems they face daily.

b) The VLE allowed for continuous updates and needed alterations in the course content; the development of discussion forums concerning teaching practices in the critical mathematical modeling process and the elaboration of questions about the pedagogical and technical aspects of this process. VLE also allowed the integration of students, tutors, and the professor to deliver comments, messages and encouragement; the conduction of pedagogical monitoring such as sending messages to all participants and participation in the discussion forums; and technical support such students and tutors access reports in the VLE. In this virtual environment, the learning occurred through socialization because knowledge was better constructed when the students worked in groups and act cooperatively in order to support and encourage each other.

Accessing Virtual Learning Environments

In the modeling process, the social environment also comes to influence learning and cognition in ways that are related to cultural context. Collaborative work via the Moodle Platform between groups of teachers, tutors, and students makes learning more effective as it generates levels of mathematical thinking by using socially and culturally relevant activities. Thus, cognition is the result of cultural artifacts in these interactions and allows for the use of *dialogical constructivism* because the knowledge source is based on the social interactions between students and teachers (Rosa & Orey, 2007).

A critical and reflective mathematical modeling environment provides concrete opportunities for students to discuss the role of mathematics as well as the nature of their models as they study systems taken from reality by using technological tools in the VLE. In accordance to this point of view, critical mathematical modeling may be understood as a language to study, understand, and comprehend problems faced community (Bassanezi, 2002). Once again, to repeat, mathematical modeling is used to analyze, simplify, and solve daily phenomena in order to predict results or modify the characteristics of these phenomena.

In this process, the purpose of critical mathematical modeling becomes the ability to develop critical skills that enable teachers and students to analyze and interpret data, to formulate and test hypotheses, and to develop and verify the effectiveness of mathematical models. In so doing, the reflections become transforming actions, seeking to reduce the degree of complexity through the choice of a system that can represent it (Rosa & Orey, 2013).

By developing strategies through a variety of technological tools, students both practice and learn to explain, understand, manage, analyze, and reflect on all parts of this system based on data. The process optimizes pedagogical conditions for teaching and learning so that students more clearly understand events around them in order to act effectively and transform phenomenon according to the needs the community.

In order to lead students towards the understanding of critical and reflective dimensions, it is necessary to expose them to a wide variety of problems or themes. As part of this process, questionings are used to explain or make predictions about the phenomena under study through the elaboration of models that represent these situations.

Final Considerations

Fundamental characteristics of teaching towards critical and reflective dimensions of mathematical modeling is the emphasis on the critical analysis of students in problems faced by a member of contemporary society. Thus, the critical perspective of students in relation to an ongoing social conditions that affect their own experiences can help them to identify common problems and collectively develop strategies to solve them (D'Ambrosio, 1990).

This paradigm incorporates a type of transformatory learning that creates conditions that help learners to challenge their worldviews and values. They are then better able to reflect critically on these experiences in order to develop data-based rational discourse by creating meanings necessary for structural transformation of society (Freire, 2000). This presents a rational transformation because it involves critical analysis of sociocultural phenomena through the elaboration of mathematical models. Mathematical modeling is therefore a teaching methodology that focuses on the development of critical-reflective efficiencies and engages students in contextualized teaching-learning processes that allowing them to get deeply and actively involved in the constructions of social significance of the world (Rosa & Orey, 2015). In short, they learn to move away from high emotional arguments, and focus on the data.

The act of creating a mathematical poem, allows for critical and reflective dimensions of mathematical modeling that are based on the comprehension and understanding of reality. When we borrow systems from reality, students begin to study them symbolically, systematically, analytically and critically. In this regard, starting from problem situations, modelers learn to make hypotheses, test them, correct them, make transfers, generalize, analyze, complete and make decisions about the object under study based less on emotion and more on data. Thus, critically reflecting about reality using mathematics becomes a transformational action that seeks to reduce complexity by allowing students to explain it, understand it, manage it and find solutions to problems that arise therein.

The study of new educational methodological proposals becomes relevant because it originates with ideas regarding social changes resulting from ongoing continuous contemporary scientific and technological developments. In order to enable teaching methods using structured learning materials and existing technological resources, it was developed the long distance learning, which refers to planned learning that normally occurs outside of school (Moore & Kearsley, 2007). On the other hand, in the last three decades, critical mathematical modeling as a teaching and learning methodology has been one of the central themes in mathematics education in Brazil and has come to offer a way to rebuild or restore what has become for many, a fragmented and meaningless mathematical knowledge. This approach appears to encourage them to develop more informed and research-based opinions in their real life.

And so it is, that we have come to consider mathematical modeling as a teaching methodology that focuses on the development of a critical and reflective efficacy that engages diverse groups of students in a contextualized teaching-learning process and that allows them to become involved in the construction of solutions of social significance (Rosa & Orey, 2007). This critical dimension of mathematical modeling is based on the comprehension and understanding of reality, in which students learn to reflect, analyze and take action on their own reality.

When we explore examples and problems from their reality, students begin to study the symbolic, systematic, analytical, and critically contexts to their work by using technological tools provided in a virtual learning environment. Because technological tools offered via the platforms are simple and functional, long distance learning modalities contribute to and greatly assist students to overcome difficulties regarding the adoption of critical mathematical modeling strategies. With discussion forums and videoconferences, professors and tutors are able to and can better analyze interactions enabled by these tools, which contributed to development of the elaboration of mathematical models in the virtual learning environment.

References Bibliographics

Alves, L. (2011). Educação a distância: conceitos e história no Brasil e no mundo. *Revista Brasileira de Aprendizagem Aberta e a Distância - RBAAD*, 10, 84-92.

Barbosa, J. (2006) Mathematical modeling in classroom: a sociocritical and discursive perspective. *ZDM, 38*(3), 293-301.

Bassanezi, R. (2002). Ensino-aprendizagem com modelagem matemática [Teaching and learning with mathematical modeling]. São Paulo, SP: Editora Contexto.

Brasil. (1998). Parâmetros curriculares nacionais (PCN): matemática [National curricular parameters: mathematics]. Brasília, DF: MEC/SEF.

Brasil. (2005). Sistema universidade aberta do Brasil – UAB [Brazilian Open University System]. Decreto nº 5.800/2006. Brasília, DF: Casa Civil.

D'Ambrosio, U. (1986). Da realidade a ação: reflexões sobre educação e matemática [From reality to action: reflections on education and mathematics]. Campinas, SP: Unicamp; São Paulo: SUMMUS.

D'Ambrosio, U. (1990). Etnomatemática [Ethnomathematics]. São Paulo, SP: Editora Ática.

Freire, P. (2000). Pedagogy of the oppressed. New York, NY: Continuum.

Freire, P. (2005). Pedagogia da autonomia: saberes necessários à pratica educativa [Pedagogy of autonomy: necessary knowledge to educational practice]. São Paulo: Paz e Terra.

Moore, M.; Kearsley, G. (2007). Handbook of distance education. Mahwah, NJ: Lawrence Erbaun Associates.

Moraes, M. (1997). *O* paradigma educacional emergente [The emergent educational paradigm]. Campinas, SP: Papirus.

Rosa, M.; Orey, D. (2007). A dimensão crítica da modelagem matemática: ensinando para a eficiência sociocrítica [The critical dimension of mathematical modeling: teaching to sociocritical efficiency]. *Horizontes*, *25*(2), 197-206.

Rosa, M.; Orey, D. (2011). Ethnomathematics: the cultural aspects of mathematics. *Revista Latinoamericana de Etnomatemática*, *4*(2), 32-54.

Rosa, M.; Orey, D. (2013). The mathematics of the curves on the wall of the colegio Arquidiocesano and its mathematical models: a case for ethnomodeling. *Journal of Mathematical Modeling and Application, 1*(8), 42-62.

Rosa, M; Orey, D. (2015). Social-critical dimension of mathematical modelling. In Gloria Ann Stillman, Werner Blum, Maria Salett Biembengut (Orgs.). *Mathematical modelling in education research and practice*: cultural, social, and cognitive influences. New York, NY: Springer, 85-395.

El *Scrapbook* como herramienta didáctica en la enseñanza de la Matemática en niveles de Secundaria

M.Sc. María Fernanda Víquez Ortiz
Ministerio de Educación Pública
mfdaviquez@gmail.com

M.Sc. Jorge Arroyo Hernández
Universidad Nacional
jorge.arroyo.hernández@una.cr

M.Sc. Evelyn Delgado Carvajal
Universidad Técnica Nacional
evelyndelgado27@gmail.com

Resumen: Los nuevos Programas de Estudio exigen el empleo de técnicas, estrategias y métodos diversos que despierten el interés y la creatividad del estudiante así como, facilitarle el logro de las habilidades propuestas y con ello la interiorización de conceptos para la aplicación inmediata. Por ello, surge la opción de implementar el Scrapbook como herramienta didáctica para el desarrollo de habilidades y la interiorización de conocimientos. En este documento, se presentan actividades para ser puestas en práctica en el aula, con su respectiva guía didáctica, para el estudio de contenidos, alcance de habilidades y competencias que favorezcan el proceso de enseñanza y por tanto, el aprendizaje del estudiante.

Palabras claves: Scrapbook, enseñanza de la matemática, técnicas de enseñanza

El Scrapbook

El Scrapbook es una técnica manual que nació para preservar recuerdos mediante la creación de álbumes. Según Palacios (2015) es el arte de armar álbumes de fotos decorados, empleando diversas técnicas, como costura, decoupage, pintura, origami, embozados y troquelados. Sin embargo, su gran auge comercial y su apreciación entre las personas por constituirse un trabajo artesanal, la ha vinculado con tarjetas de felicitación, invitaciones, colillas, cajas y bolsas de regalo, decoración de libretas, cuadernos, entre otros. Para Alonso (2013), el Scrapbook es una técnica que consiste en personalizar fotografías, narrar una historia empleando accesorios tales como cartulinas, papales decorados, recortes, cintas, remaches y botones.

El Scrapbook también conocido como Scrapbooking o scrap emplea diversos materiales, algunos muy particulares y otros que van desde el reuso o reciclaje, ello según la imaginación de la persona que crea. Para Palacios (2015), existen materiales básicos como papeles, cartulinas decoradas, adhesivos (gomas líquidas, de barra, cintas), tijeras, guillotinas y huesos. Sin embargo, existen infinidad de materiales y herramientas que se han ido lanzando al mercado para facilitar el trabajo, como perforadores, máquinas de corte y embozado, secadoras, polvos embosadores, tintas, washi tape, encuadernadoras, sellos, lapiceros de gel, tablas de corte y muchas otras.

La matemática en el Scrapbook

El Scrapbook requiere de conversiones de medidas (pulgadas a centímetros), uso de fracciones, creación de figuras semejantes, entre muchos otros. De esta manera, se puede afirmar que el Scrapbook puede ser empleado como técnica para la enseñanza de la Matemática, ya que promueve que el estudiante sea generador de su propio conocimiento y se constituya como promotor y agente activo en todo momento. Asimismo, se puede afirmar que esta técnica le proporciona al estudiante condiciones oportunas para que desarrolle capacidades y alcance con éxito, el logro de los objetivos.
Por otra parte, esta técnica didáctica permite implementar actividades que enlazan lo concreto con lo abstracto y de esta manera la construcción del conocimiento, uno de los fines de los Programas de Estudio actuales, donde además se menciona que se deben adoptar diferentes ejes disciplinares, entre ellos: la contextualización activa como componente pedagógico especial y la potenciación de actitudes y creencias positivas en torno a las Matemáticas (MEP, 2012). En esta línea, Muñoz (2013) asegura que el docente debe estar en la capacidad de facilitar la comprensión de los contenidos, esto requiere partir de lo concreto hacia lo abstracto, proporcionando situaciones reales, materiales manipulativos para que experimenten, indaguen, conjeturen y argumenten, lo que se puede alcanzar a través del Scrapbook.

El Scrapbook requiere la aplicación en todo momento de la Matemática, por ejemplo:

- De *Números* para crear una tarjeta, para cortar sus piezas de manera que éstas se ajusten a los tamaños, realizar conversiones de centímetros a pulgadas, de decímetros a centímetros o según las medidas proporcionadas en los tutoriales.
- De la *Geometría*, al crear piezas semejantes se requiere dicha noción, se aborda el concepto de homotecias con razones de uno o distintas. Se contemplan otros conceptos geométricos al doblar un papel, especificar su largo y ancho, diagonal, relaciones entre radio y lado o lado y apotema, entre otros. Al estudiar la geometría del espacio, permite identificar diferentes sólidos y sus elementos: caras laterales y basales, aristas, apotemas, alturas, vértices y otros, además de estudiar sus áreas.
- De *Relaciones y Álgebra*, al constituir modelos matemáticos para calcular áreas, para establecer relaciones entre altura, ancho y largo.
- De *Estadística y Probabilidad*, al proporcionar la posibilidad de definir el concepto de frecuencia, moda, espacio muestral, cálculo de probabilidad simple, eventos mutuamente excluyentes, no excluyentes, eventos probables, seguros e imposibles.

Conclusiones

El Scrapbook es una técnica útil en el proceso de enseñanza de la Matemática, puesto que le permite al estudiante vincular los conceptos abstractos con material concreto o recursos tangibles: al manipular cartulina, medir, relacionar medidas y llegar a abstracciones, desarrollar habilidades cognitivas de forma dinámica, trabajar y mejorar la motora fina, desarrollar la percepción espacial así como la precisión manual y el trabajo colaborativo, desarrollar la visualización, la manipulación y la descripción de figuras geométricas, asimilar de diferentes conceptos estadísticos, la noción de probabilidad y relacionar las figuras concretas con el álgebra.

Para el docente, el Scrapbook resulta una herramienta valiosa al proporcionarle una manera de hacer de sus clases espacios más dinámicos y no rutinarios y motivar al estudiante a ser creativo, desarrollar sus propios modelos de manera que vincule la Matemática con expresiones artísticas.

En síntesis, las actividades empleando el Scrapbook despiertan la criticidad del estudiante, de forma creativa, para posteriormente representar los conceptos hasta alcanzar un razonamiento lógico matemático que sirva de argumento y sustento en la toma de decisiones.

Por tanto, la finalidad de este trabajo es invitar a los docentes a aplicar y plantear actividades apoyadas en el Scrapbook en concordancia con los Programas de Estudio actuales para que favorezcan el aprendizaje de los jóvenes y adultos en sus aulas.

Referencias bibliográficas

Alonso, J. (2013). Misión: Scrapbooking (libro de recortes). Aguilar, España. Recuperado de http://www.librosaguilar.com/uploads/ficheros/libro/primeras-paginas/201304/primeras-paginas-mision-scrapbooking-libro-recortes.pdf

Ministerio de Educación Pública. (2012). Programas de Estudio: Educación General Básica y Ciclo Diversificado. San José, Costa Rica.

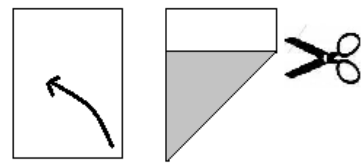
Muñoz, C. (2013). Los materiales en el aprendizaje de las matemáticas. Universidad de la Rioja. Recuperado de http://biblioteca.unirioja.es/tfe_e/TFE000754.pdf

Palacios, C. (2015). ¿Qué es el Scrapbooking? *Revista Ohlalá*. Recuperado de http://www.revistaohlala.com/1621070-que-es-el-scrapbooking

ACTIVIDAD #1. Del corazón a la flecha

Polígonos	**1. Calcular perímetros y áreas de polígonos no regulares utilizando un sistema de coordenadas rectangulares** **2. Resolver problemas que involucren polígonos y sus diversos elementos**

GUIA PARA EL ESTUDIANTE



1. Tome la lámina rectangular, dóblela llevando la punta derecha inferior al borde contrario de la hoja, tal y como muestra la figura. Corte el sobrante

2. Al abrirlo, ¿Qué figura geométrica obtiene?______________

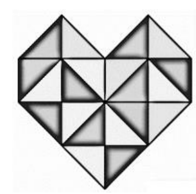
3. ¿Cómo son sus lados? _________________

4. ¿Cómo son sus ángulos?_________________

5. El doblez que tiene su figura recibe el nombre de DIAGONAL. Córtelo sobre esta diagonal. ¿Qué figuras obtuvo?__________________

6. ¿Cómo se clasifica según sus ángulos?________________

7. Mida sus catetos y anote sus medidas: ________________



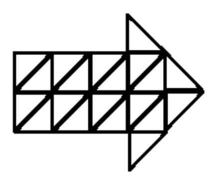
8. Determine la medida de su hipotenusa y anótela:________________

9. Formaremos un corazón como el adjunto, en el piso. Espere su turno, para ir a colocarlo.

10. Sabiendo cuanto miden sus catetos, ¿Cuál es el área de cada triangulo?_______________

11. ¿Cuál es el área de todo el corazón?__________________________

12. ¿Cuál es el perímetro del corazón?__________________________



13. Usando la misma cantidad de piezas, construya la siguiente flecha

14. ¿Cuál es el área de toda la flecha?__________________________

15. ¿Cuál es el perímetro de la flecha?__________________________

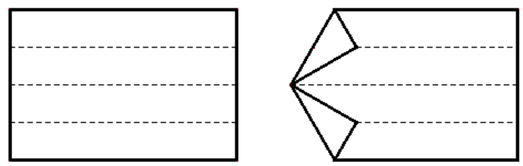
16. ¿Cuál varió, el perímetro o el área? ¿por qué?____

ACTIVIDAD #2. La tarjeta hexagonal

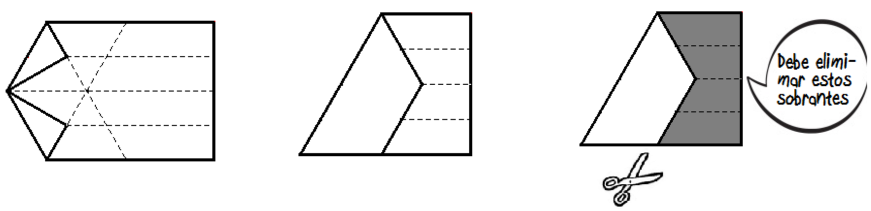
Polígonos	1. **Determinar las medidas de los ángulos internos y externos de polígonos en diferentes contextos** 2. **Determinar la medida de perímetros y áreas de polígonos en diferentes contextos**

Siga las siguientes instrucciones, guíese con las ilustraciones:

1. Tome el rectángulo de papel, dóblelo (a lo largo) a la mitad y nuevamente a la mitad (es decir, en 4 secciones congruente), posteriormente realice dos pestañas tal y como se observa en la figura



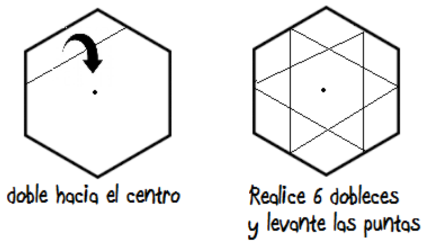
2. Luego doble nuevamente, suponiendo la prolongación de una de la pestañas, como se observa a continuación.



3. Corte el excedente.

- ¿Qué figura obtuvo?_____________________
- ¿Cómo son sus lados entre sí?_________________________
- ¿Cuánto medirá su ángulo central? _________________ ¿Cómo lo puede justificar?_______________
- ¿Cómo son sus ángulos internos entre sí?___________________ Realice los dobleces necesarios para determinar la medida de su ángulo interno. ¿Cuál es la medida de este?________________
- Doble ese polígono formando 6 triángulos. ¿Cómo se clasifican esos triángulos?____________
- Sabiendo que el hexágono regular está conformado por 6 triángulos equiláteros ¿Cómo determina usted el área de cada uno de esos triángulos?_____________________
- ¿Cuál es el área de TODO el hexágono? ¿Cuál es su fórmula?_____________________________
- Observe el radio, el lado y la apotema. ¿Qué relación existe entre ellos?______________

4. Construyamos la tarjeta



Luego, empuje las puntas formando 6 "picos".

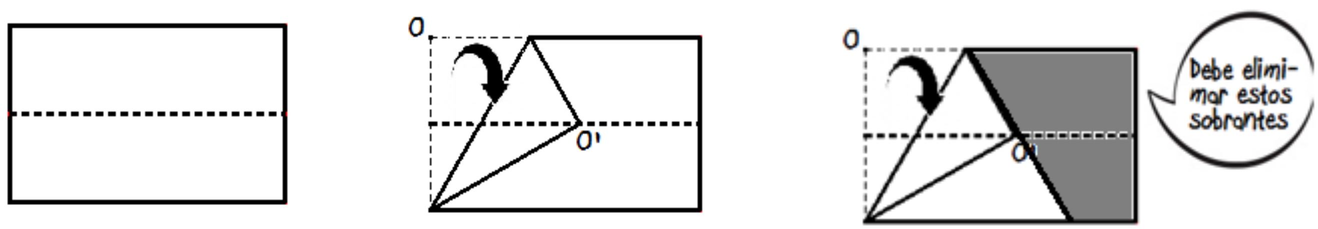
Compare el área del primer hexágono construido con la parte de atrás de su tarjeta

ACTIVIDAD #3. La cajita de regalo: de un triángulo a una pirámide

Visualización espacial	**1. Identificar la base, las caras laterales, la altura, las apotemas y el ápice o cúspide de una pirámide.** **2. Identificar las caras laterales, las bases y la altura de un prisma recto**

Siga las siguientes instrucciones, guíese con las ilustraciones**:**

1. Doble a la mitad (a lo largo del rectángulo), luego ubique el O sobre la paralela media (O´)



2. ¿Qué nombre recibe la figura resultante? ___________________________________
3. ¿Cómo son sus lados?____
4. ¿Cómo son sus ángulos?__________________________ ¿Qué medida tendrán?______
5. Formemos 4 triángulos congruentes y a partir de ellos, construyamos una cajita de regalo.
6. ¿Qué cuerpo geométrico se formó?________________________
7. Se llama APOTEMA DE LA PIRAMIDE a la altura de cada uno de esos triángulos, coloque Washi tape a uno de ellos
8. Mantenga la cajita cerrada. Cada "filito de la cajita" recibe el nombre de ARISTA. ¿Cuántas aristas tiene?_____________
9. Cada triángulo pequeño resultante se llama CARA. ¿Cuántas caras tiene?________________
10. Decore cada cara LATERAL. ¿Cuántas caras laterales tiene?________
11. Deje sin decorar las caras basales. ¿Cuántas caras basales tiene?________
12. ¿Qué cree usted que es la Cúspide o ápice de la pirámide?_____________________________
13. ¿Es la apotema de la pirámide lo mismo que la altura de ella?___________________________

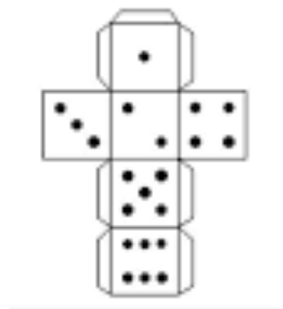
14. Defina apotema de la pirámide:____

15. Defina altura de la pirámide:____

ACTIVIDAD #4. Un dado

Visualización espacial	**1. Reconocer en figuras tridimensionales diversos elementos como caras, aristas y vértices** **2. Establecer relaciones entre los elementos de figuras tridimensionales: vértices, caras y aristas, rectas y segmentos paralelos, perpendiculares, planos paralelos y perpendiculares**

El siguiente dibujo corresponde a un cubo sin armar



Conteste lo que se le solicita

1. ¿Cuántos vértices tiene el dado?______________
2. ¿Cuántas aristas tiene el dado?_______________
3. ¿Cuántas caras tiene el dado?________________
4. ¿Cuántas arista coinciden en un mismo vértice?_________________
5. ¿Comparten la cara 1 y 6 una arista?____________
6. ¿Comparten la cara 3 y 4 una arista?____________
7. ¿El plano del 1 y del 5 son paralelos?___________
8. ¿El plano del 3 y del 5 son perpendiculares?__________________
9. ¿El plano del 1 y del 6 son perpendiculares?__________________

- Recorte el cubo que se le proporciona en la cartulina
- Numérelo tal y como se observa en la imagen.
- Ármelo y péguelo
- Corrobore sus respuestas

ACTIVIDAD #5. Vistiendo personajes

Espacio Muestral **Probabilidad**	1. **Identificar el espacio muestral y sus puntos muestrales como resultados simples en una situación o experimento aleatorio de representarlo por medio de numeración de elementos o de diagramas.** 2. **Determinar la probabilidad de un evento como la razón entre el número de resultados favorables entre el número total de resultados.**

1. Vista la muñeca que usted tiene en su separador de libros, empleando la ropa que se le proporciona.
2. No mire lo que hizo su compañero de al lado, anote las posibles opciones de vestir la muñeca

   ____

   ____

   ____

3. ¿Cuántos pantalones y enaguas hay?______
4. ¿Cuántas blusas hay?_________
5. ¿Cuántas posibilidades hay de vestir a la muñeca?___________
6. ¿Existe una manera o fórmula de saber cuántas veces se pueden combinar las prendas?_______

¿Cuál?____

Las combinaciones posibles se llaman ESPACIO MUESTRAL.

La <u>probabilidad simple</u> es la razón entre el número de resultados favorables por el número total de resultados.

7. ¿Qué probabilidad hay de elegir un pantalón café, de entre las prendas, al azar?_____
8. ¿Qué probabilidad hay de elegir una blusa beige, de entre todas las prendas, al azar?_____
9. Suponga que su blusa fucsia se rompió. ¿Cuál es la probabilidad de elegir, al azar, una blusa de entre todas las prendas?_________
10. ¿Existe la posibilidad de que algunos de sus demás compañeros tenga la muñeca mudada igual que usted?___________ ¿Por qué?____